

**Section-I: General Ability**  
**Q. No. 1 - 5 Carry One Mark Each**

1. Until Iran come along. India had never been \_\_\_\_\_ in kabaddi.  
 (A) defeated (B) defeating  
 (C) defeat (D) defeatist

**Key: (A)**

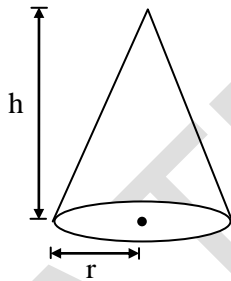
2. The fishermen, \_\_\_\_\_ the flood victims owed their lives, were rewarded by the government  
 (A) whom (B) to which (C) to whom (D) that

**Key: (C)**

3. The radius as well as the height of a circular cone is increases by 10%. The percentage increase in its volume is \_\_\_\_\_.  
 (A) 17.1 (B) 21.0 (C) 33.1 (D) 72.8

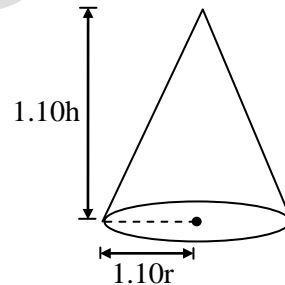
**Key: (C)**

**Sol:** Initial Stage After increasing 10% for radius & height



Volume of circular cone,

$$V_i = \frac{1}{3} \pi r^2 h$$



$$\begin{aligned} \text{Volume, } V_f &= \frac{1}{3} \pi (1.1r)^2 (1.1h) \\ &= \frac{1.331}{3} \pi r^2 h \end{aligned}$$

$$\text{Percentage increase} = \frac{V_f - V_i}{V_i} \times 100 = \frac{\frac{1.331}{3} \pi r^2 h - \frac{1}{3} \pi r^2 h}{\frac{1}{3} \pi r^2 h} \times 100 = \frac{1.331 - 1}{1} \times 100 = 33.1\%$$

4. Five numbers 10, 7, 5, 4, 2 are arranged in a sequence from left to right following the directions given below:  
 (1) No two odd or even numbers are next to each other.  
 (2) The second number from left is exactly half of the left -most number.  
 (3) The middle number is exactly twice the right most number.

Which is the second number from the right ?

- (A) 2                      (B) 4                      (C) 7                      (D) 10

**Key:** (C)

**Sol:** Numbers are 2, 4, 5, 7, and 10

The correct order of arrangement 10, 5, 4, 7, and 2

Thus an arrangement follows given three conditions

Second number from right = 7

5. "Some students were not involved in the strike". If the above statement is true, which of the following conclusions is/are logically necessary ?

1. Some who were involved in strike were students.
2. No student was involved in the strike.
3. At least one student was involved in the strike.
4. Some who were not involved in the strike were students.

- (A) 1 and 2                      (B) 3                      (C) 4                      (D) 2 and 3

**Key:** (C)

**Q. No. 6 - 10 Carry Two Marks Each**

6. "I read somewhere that in ancient times the prestige of a kingdom depended upon the number of taxes that it was able to levy on its people. It was very much like the prestige of a head-hunter in his own community".

Based on the paragraph above, the prestige of a head-hunter depended upon \_\_\_\_

- (A) the prestige of the kingdom  
(B) the prestige of the heads  
(C) the number of taxes he could levy  
(D) the number of head she could gather

**Key:** (D)

7. Two trains started at 7 AM from the same point. The first train travelled towards north at a speed of 80 km/h and the second train travelled south at a speed of 100 km/h. The time at which they were 540 km apart is \_\_\_\_\_ AM.

- (A) 9                      (B) 10                      (C) 11                      (D) 11:30

**Key:** (B)

**Sol:** For X

Time taken = t

Distance  $x = \text{velocity} \times \text{time}$

$$x = 80t \Rightarrow t = \frac{x}{80} \quad \dots(1)$$

For  $y$  time taken =  $t$

$$\text{Distance } y = 100t \Rightarrow t = \frac{y}{100} \quad \dots(2)$$

$$x + y = 540\text{km} \quad \dots(3)$$

From (1) and (2)

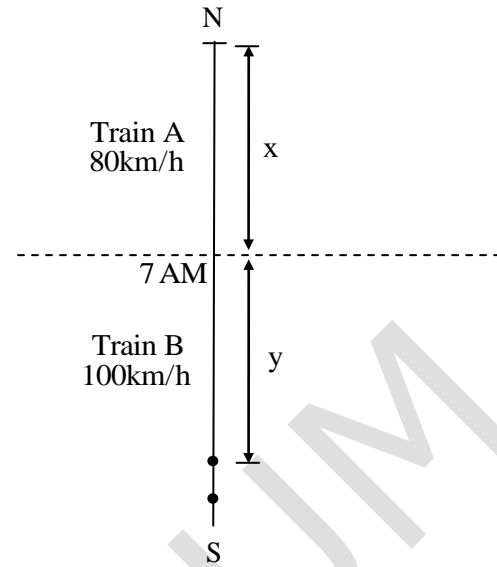
$$t = \frac{x}{80} = \frac{y}{100} \Rightarrow x = 0.8y$$

$$x + y = 540 \Rightarrow 0.8y + y = 540$$

$$1.8y = 540 \Rightarrow y = 300\text{km}$$

$$\text{Time taken} = \frac{y}{100} = \frac{300}{100} = 3\text{hrs}$$

Time at which these trains = 7.00AM + 3hrs = 10.00AM



8. In a country of 1400 million population, 70% own mobile phones. Among the mobile phone owners only 294 million access the internet. Among these Internet users, only half buy goods from e-commerce portals. What is the percentage of these buyers in the country ?
- (A) 10.50                      (B) 14.70                      (C) 15.00                      (D) 50.00

**Key:** (A)

**Sol:** Total population = 1400 million

Number of people whose having own mobile phones

$$= 70\% \text{ of } 1400 = 0.7 \times 1400 = 980 \text{ million}$$

Number of people whose accesses the internet = 294 million

Number of people who buy goods from e-commercial portals = Half of internet users

$$= \frac{294}{2} = 147 \text{ million}$$

$$\text{Percentage buyers} = \frac{147 \text{ million}}{1400 \text{ million}} \times 100 = 10.5\%$$

9. The nomenclature of Hindustani music has changed over centuries. Since the medieval period *dhrupad* styles were identified as *baanis*. Terms like *gayaki* and *baaj* were used to refer to vocal and instrumental styles, respectively. With institutionalization of music education the term *gharana* became acceptable. *Gharana* originally referred to hereditary musicians from a

particular lineage, including disciples and ground disciples. Which one of the following pairings is NOT Correct ?

- (A) Dhruvad, baani (B) Gayaki, Vocal  
(C) Baaj, institution (D) Gharana, lineage

**Key: (C)**

**10.** Since the last one year, after a 125 basis point reduction in repo rate by the Reserve Bank of India, banking institutions have been making a demand to reduce interest rates on small savings schemes. Finally, the government announced yesterday a reduction in interest rates on small saving schemes to bring them on par with fixed deposit interest rates.

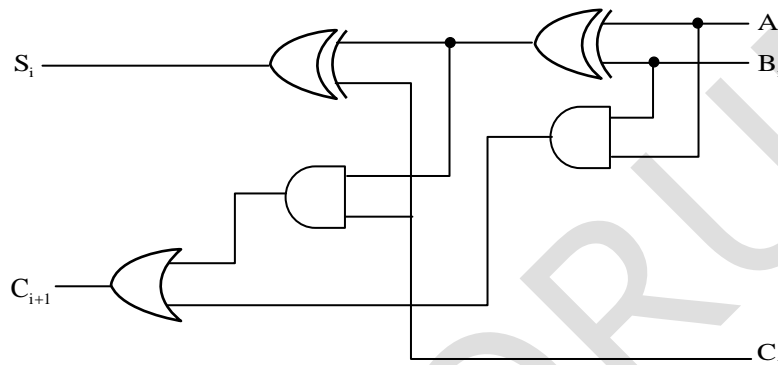
Which one of the following statements can be inferred from the given passage ?

- (A) Whenever the Reserve Bank of India reduces the repo rate, the interest rates on small saving schemes are also reduced.  
(B) Interest rates on small saving schemes are always maintained on par with fixed deposit interest rates.  
(C) The government sometimes takes into consideration the demands of banking institutions before reducing the interest rates on small saving schemes.  
(D) A reduction in interest rates on small saving schemes follow only after a reduction in repo rate by the Reserve Bank of India.

**Key: (D)**

**Section-II: Instrumentation Engineering**  
**Q. No. 1 to 25 Carry One Mark Each**

1. The figure below shows the  $i^{\text{th}}$  full-adder block of a binary adder circuit.  $C_i$  is the input carry and  $C_{i+1}$  is the output carry of the circuit. Assume that each logic gate has a delay of 2 nanosecond, with no additional time delay due to the interconnecting wires. If the inputs  $A_i, B_i$  are available and stable throughout the carry propagation, the maximum time taken for an input  $C_i$  to produce a steady-state output  $C_{i+1}$  is \_\_\_\_\_ nanosecond.



**Key: (6)**

**Sol:** To get circuit output bit, the input signals has to propagate through 3 stages and each stage having delay  $2n$  sec so the total delay is  
 $2 \times 3 = 6n$  sec.

2. The resistance of a resistor is measured using a voltmeter and an ammeter. The voltage measurements have a mean value of 1V and standard deviation of 0.12V while current measurements have a mean value of 1 mA with standard deviation of 0.05mA. Assuming that the errors in voltage and current measurements are independent, the standard deviation of the calculated resistance value is \_\_\_\_\_  $\Omega$ .

**Key: (130)**

**Sol:** Given  $V_{\text{mean}} = 1V, \sigma_v = 0.12V$   
 $I_{\text{mean}} = 1mA, \sigma_I = 0.05mA$

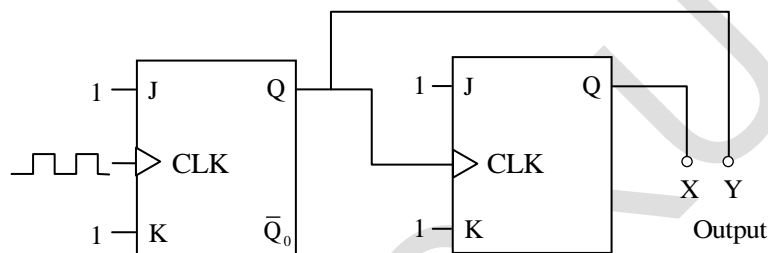
$$R = \frac{V}{I} \text{ (from basic)}$$

$$\sigma_R = \sqrt{\left(\frac{\partial R}{\partial I}\right)^2 (\sigma_I)^2 + \left(\frac{\partial R}{\partial V}\right)^2 \sigma_V^2} = \sqrt{\left(\frac{-V}{I^2}\right)^2 \sigma_I^2 + \left(\frac{1}{I}\right)^2 \sigma_V^2}$$

$$= \sqrt{\left(\frac{-1}{10^{-3}}\right)^2 (0.05 \times 10^{-3})^2 + \left(\frac{1}{10^{-3}}\right)^2 \times 0.12^2} = \sqrt{2500 + 14400}$$

$\sigma_R = 130\Omega \rightarrow$  It is standard deviation in calculated resistance.

3. The circuit shown in the figure below uses ideal positive edge-triggered synchronous J-K flip flops with outputs X and Y. If the initial state of the output is  $X = 0$  and  $Y = 0$  just before the arrival of the first clock pulse, the state of the output just before the arrival of the second pulse is



- (A)  $X=0, Y=0$       (B)  $X=0, Y=1$       (C)  $X=1, Y=0$       (D)  $X=1, Y=1$

**Key:** (D)

**Sol:** Just before 2<sup>nd</sup> clock means, after the 1<sup>st</sup> clock, since the given circuit is a standard 2 bit ripple down counter, the counting sequence is

00-11-10-01-00-11.....

So if the initial state is  $X=0, Y=0$ , then after 1<sup>st</sup> clock  $X=1, Y=1$

4. The input  $x[n]$  and output  $y[n]$  of discrete-time system are related as  $y[n] = \alpha y[n-1] + x[n]$ . The condition of  $\alpha$  for which the system is Bounded-Input Bounded-Output (BIBO) stable is

- (A)  $|\alpha| < 1$       (B)  $|\alpha| = 1$       (C)  $|\alpha| > 1$       (D)  $|\alpha| < 3/2$

**Key:** (A)

**Sol:**  $y(n) = \alpha y(n-1) + x(n)$   
 $\Rightarrow y(z)(1 - \alpha z^{-1}) = x(z)$   
 $\Rightarrow \frac{y(z)}{x(z)} = \frac{1}{1 - \alpha z^{-1}} \Rightarrow H(z) = \frac{1}{1 - \alpha z^{-1}}$   
 $\Rightarrow h(n) = \alpha^n u(n)$

→The system can be stable if  $\sum_{K=-\infty}^{\infty} |h(K)| < \infty$  to satisfy this condition  $|\alpha| < 1$  i.e

$$\sum_{K=0}^{\infty} \alpha^k = \frac{1}{1-\alpha}, \text{ Provided } |\alpha| < 1.$$

5. A box has 8 red balls and 8 green balls. Two balls are drawn randomly in succession from the box without replacement. The probability that the first ball drawn is red and the second ball drawn is green is

(A) 4/15                      (B) 7/16                      (C) 1/2                      (D) 8/15

**Key:** (A)

**Sol:** Total balls = 8(red) + 8 (green) =16

Let A, B be events respectively that drawing a red ball in 1<sup>st</sup> draw & a green ball in 2<sup>nd</sup> draw and balls drawn without replacement.

$$\begin{aligned} \therefore \text{Required probability is } P(A \cap B) &= P(A) \cdot P(B/A) \\ &= \frac{8C_1}{16C_1} \times \frac{8C_1}{15C_1} = \frac{1}{2} \times \frac{8}{15} = \frac{4}{15} \end{aligned}$$

6. Consider a circuit comprising only resistors with constant resistance and ideal independent DC voltage sources. If all the resistances are scaled down by a factor 10, and all source voltages are scaled up by a factor 10, the power dissipated in the circuit scales up by a factor of \_\_\_\_\_.

**Key:** (1000)

**Sol:**  $P = \frac{V^2}{R}$

$$P' = \frac{V^2}{R'} = \frac{(10V)^2}{(R/10)} = 1000 \frac{V^2}{R}$$

$$\frac{P'}{P} = 1000$$

$$\Rightarrow P' = 1000 P \Rightarrow P_{old} = 1000 P_{new}$$

So Power increased by 1000 times.

7. If each of the values of inductance, capacitance and resistance of a series LCR circuit are doubled, the Q-factor of the circuit would

(A) reduce by a factor  $\sqrt{2}$                       (B) reduce by a factor 2  
(C) increase by a factor  $\sqrt{2}$                       (D) increase by a factor 2

**Key: (B)**

**Sol:** → For a series RLC circuit

$$Q = \frac{1}{R} \sqrt{\frac{L}{C}} \quad \dots(1)$$

→ If all components values becomes doubles

$$Q' = \frac{1}{2R} \sqrt{\frac{2L}{2C}} = \frac{1}{2} \left[ \frac{1}{R} \sqrt{\frac{L}{C}} \right] \quad \dots(2)$$

→ By Comparing equation (1) and equation (2) we can say the new value of quality factor becomes half of old quality factor.

8. A signal  $\cos(2\pi f_m t)$  modulates a carrier  $\cos(2\pi f_c t)$  using the double-sideband-with-carrier (DSBWC) scheme to yield a modulated signal  $\cos(2\pi f_c t) + 0.3\cos(2\pi f_m t)\cos(2\pi f_c t)$ . The modulation index is \_\_\_\_\_. (Rounded off to one decimal place).

**Key: (0.3)**

**Sol:** → Standard form of DSB with carrier modulation is given by

$$A_c [1 + \mu A_m \cos 2\pi f_m t] \cos 2\pi f_c t$$

→ In the given problem  $A_c = A_m = 1$ , So the equation becomes

$$[1 + \mu \cos 2\pi f_m t] \cos 2\pi f_c t$$

→ Then given modulated signal is

$$S(t) = \cos 2\pi f_c t + 0.3 \cos 2\pi f_m t \cos 2\pi f_c t$$

$$S(t) = [1 + 0.3 \cos 2\pi f_m t] \cos 2\pi f_c t$$

By comparing with standard equation

$$[1 + \mu \cos 2\pi f_m t] \cos 2\pi f_c t$$

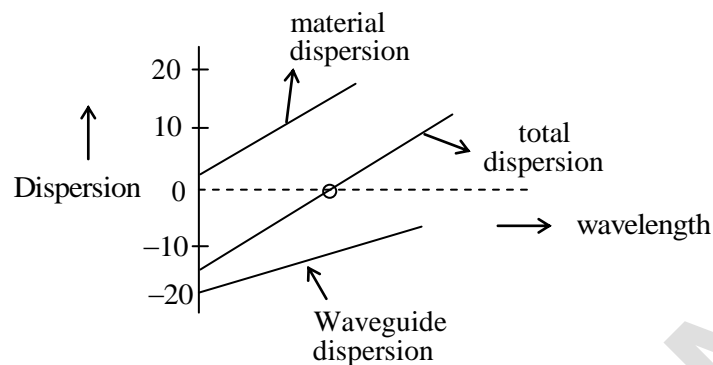
$$\mu = 0.3$$

9. In a single-mode optical fiber, the zero-dispersion wavelength refers to the wavelength at which the
- (A) material dispersion is zero
  - (B) waveguide dispersion is zero
  - (C) sum of material dispersion and waveguide dispersion is zero
  - (D) material dispersion and waveguide dispersion are simultaneously zero.



**Key: (C)**

**Sol:** For a single mode optical fibre, the wavelength  $V_s$  Dispersion curve is given by



So, total dispersion is sum of waveguide and material dispersion.

10. The total number of Boolean functions with distinct truth-tables that can be defined over 3 Boolean variables is \_\_\_\_\_.

**Key: (256)**

**Sol:** Using 'n' number of Boolean variable we can have  $2^{2^n}$  number of Boolean function so when  $n=3$  it becomes  $2^{2^3} = 2^8 = 256$

11. A  $3 \times 3$  matrix has eigen values 1, 2 and 5. The determinant of the matrix is \_\_\_\_\_.

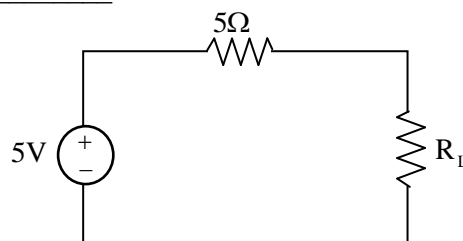
**Key: (10)**

**Sol:** Determinant of a matrix 0 is given by product of its eigen values.

$$|A| = \lambda_1 \lambda_2 \lambda_3 = 1 \times 2 \times 5 = 10$$

12. In the circuit shown below, maximum power is transferred to the load resistance  $R_L$ , when

$$R_L = \text{_____} \Omega .$$



**Key: (5)**

**Sol:** By maximum power transfer theorem, if a variable resistor is connected across a practical voltage sources then the load  $R_L$  receives maximum power from source when

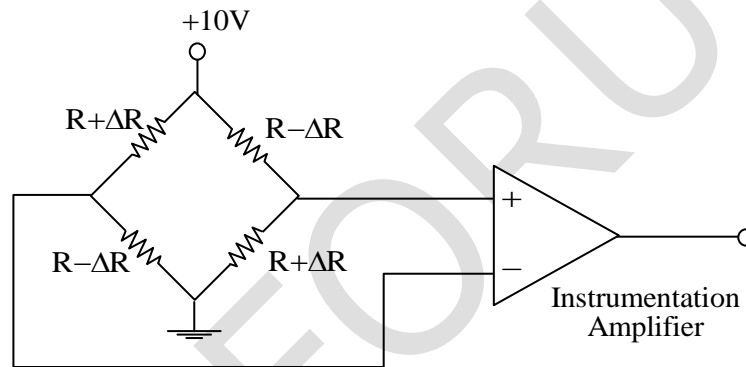
$$R_L = R_s \Rightarrow R_L = 5\Omega.$$

13. An 8-bit weighted resistor digital-to-analog converter (DAC) has the smallest resistance of 500 Ω. The largest resistance has a value of \_\_\_\_\_ kΩ .

**Key:** (64)

**Sol:** → In a n bit binary weighted resistor type DAC, the minimum values of resistance is  $2^0 R = R = 500$  maximum value of resistance is  $2^{n-1} R$   
→ So if n=8 then maximum value of resistance is  
 $2^{8-1} R = 2^7 R = 128R = 128 \times 500 = 64000 \Omega = 64 \text{ k}\Omega$

14. Four strain gauges in a Wheatstone bridge configuration are connected to an instrumentation amplifier as shown in the figure. From the choices given below, the preferred value for the common mode rejection ratio (CMRR) of the amplifier, in dB, would be



- (A) -20                      (B) 0                      (C) 3                      (D) 100

**Key:** (D)

15. The correct biasing conditions for typical operation of light emitting diodes, photodiodes, Zener diodes are respectively,

- (A) forward bias, reverse bias, reverse bias  
(B) reverse bias, reverse bias, forward bias  
(C) forward bias, forward bias, reverse bias  
(D) reverse bias, forward bias, reverse bias

**Key:** (A)

| <b>Sol:</b> | <u>Type of Diode</u>         | → | <u>Biasing</u> |
|-------------|------------------------------|---|----------------|
| i.          | Light Emitting diodes (LEDs) | → | Forward bias   |
| ii.         | Photo diodes                 | → | Reverse bias   |
| iii.        | Zener diodes                 | → | Reverse bias   |

16. The output  $y(t)$  of a system is related to its input  $x(t)$  as

$y(t) = \int_0^t x(\tau - 2) d\tau$ . where,  $x(t) = 0$  and  $y(t) = 0$  for  $t \leq 0$ . The transfer function of the system is

- (A)  $\frac{1}{s}$                       (B)  $\frac{(1 - e^{-2s})}{s}$                       (C)  $\frac{e^{-2s}}{s}$                       (D)  $\frac{1}{s} - e^{-2s}$

**Key:** (C)

**Sol:**  $y(t) = \int_0^t x(\tau - 1) dz$

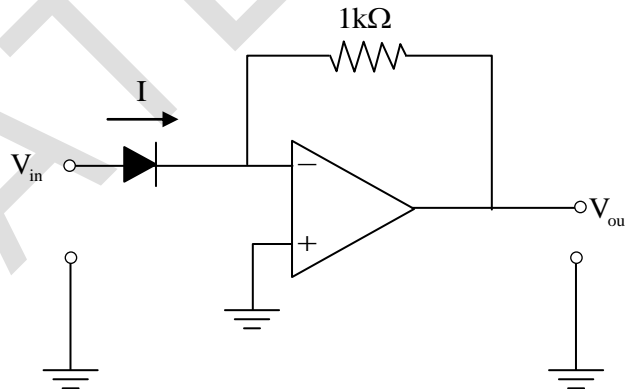
$\rightarrow x(\tau) \leftrightarrow x(s)$

$x(\tau - 2) \leftrightarrow x(s)e^{-2s}$  ( $\because$  time shift properly)

$\int_0^t x(\tau - 2) dz \leftrightarrow \frac{x(s)e^{-2s}}{s}$

$\Rightarrow Y(s) = \frac{x(s)e^{-2s}}{s} \Rightarrow \frac{Y(s)}{X(s)} = 1 + (s) = \frac{e^{-2s}}{s}$

17. In the circuit shown below, the input voltage  $V_{in}$  is positive. The current (I) – Voltage (V) characteristics of the diode can be assumed to be  $I = I_0 e^{V/V_T}$  under the forward bias condition, where  $V_T$  is the thermal voltage and  $I_0$  is the reverse saturation current. Assuming an ideal op-amp, the output voltage  $V_{out}$  of the circuit is proportional to



- (A)  $\log_e (V_{in} / V_T)$                       (B)  $2V_{in}$   
(C)  $e^{V_{in}/V_T}$                       (D)  $V_{in}^2$

**Key:** (C)

**Sol:**  $V_{out} = 0 - IR$   
 $= -IR \quad \dots(1)$   
 $+V_{in} - V_D = 0 \Rightarrow V_D = V_{in}$

Diode current equation is given by

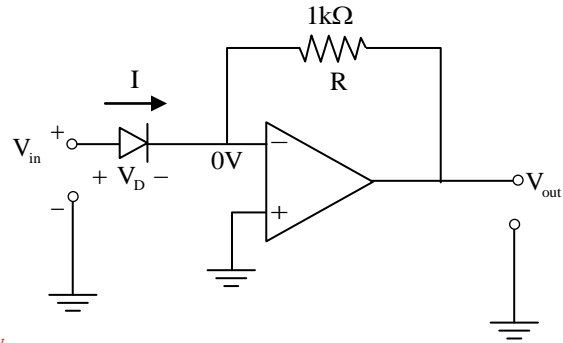
$$I = I_0 \exp \frac{V_D}{V_T} = I_0 e^{\frac{V_{in}}{V_T}} \quad (\because V_D = V_{in})$$

Substitute the value of I in equation (1), then we get

$$V_{out} = -I_0 e^{\frac{V_{in}}{V_T}} \times 1 \times 10^3 \text{ Volt}$$

From above equation it is obvious that  $V_{out} \propto e^{\frac{V_{in}}{V_T}}$

Given circuit is a prototype of exponential (Antilogarithmic) amplifier.



18.  $\vec{a}$ ,  $\vec{b}$ ,  $\vec{c}$  are three orthogonal vectors. Given that  $\vec{a} = \hat{i} + 2\hat{j} + 5\hat{k}$  and  $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$ , the vector  $\vec{c}$  is parallel to
- (A)  $\hat{i} + 2\hat{j} + 3\hat{k}$       (B)  $2\hat{i} + \hat{j}$       (C)  $2\hat{i} - \hat{j}$       (D)  $4\hat{k}$

**Key:** (C)

**Sol:** Let  $\vec{c}$  be parallel to a vector  $\vec{d} \Rightarrow \vec{a}, \vec{b}, \vec{d}$  are also mutually perpendicular

$\Rightarrow$  Dot product of every two vectors as 0.

By option method, the vector say  $\vec{d}$  in option (C) only satisfies  $\vec{a} \cdot \vec{d} = 0$  and  $\vec{b} \cdot \vec{d} = 0$

Hence answer is option (C)

19. The vector function  $\vec{A}$  is given by  $\vec{A} = \vec{\nabla}u$ , where  $u(x,y)$  is a scalar function. Then  $|\vec{\nabla} \times \vec{A}|$  is
- (A) -1      (B) 0      (C) 1      (D)  $\infty$

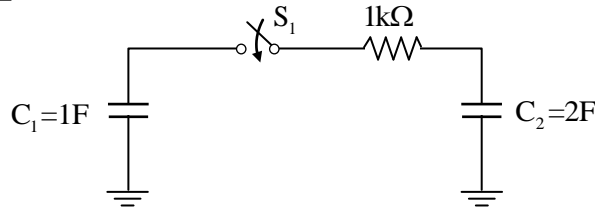
**Key:** (B)

**Sol:**  $\nabla \times \vec{A} = \nabla \times (\nabla u) = \vec{0}$  ie., Zero vector

$\therefore |\nabla \times \vec{A}| = 0 \Rightarrow$  option (b)

(From vector Identities  $\text{curl}(\text{grad } \phi) = \vec{0}$ , where  $\phi$  is scalar function).

20. In the circuit shown below, initially the switch  $S_1$  is open, the capacitor  $C_1$  has a charge of 6 coulomb, and the capacitor  $C_2$  has 0 coulomb. After  $S_1$  is closed, the charge on  $C_2$  in steady state is \_\_\_\_\_ coulomb.



**Key: (4)**

**Sol: Method-I:** For  $t > 0$ , the circuit is,

$$I(s) = \frac{6/s}{(3+2000s)} = \frac{6 \times 2}{(3+2000s) \cdot 2s}$$

$$V_{C_2}(s) = \frac{12}{(3+2000s)} \times \frac{1}{2s}$$

From final value theorem,

$$V_{C_2}(t = \infty) = V_{C_2}(0) = \lim_{s \rightarrow 0} \frac{12s}{(3+2000 \times D)} \times \frac{1}{2s} = \frac{12}{3 \times 2} = 2V$$

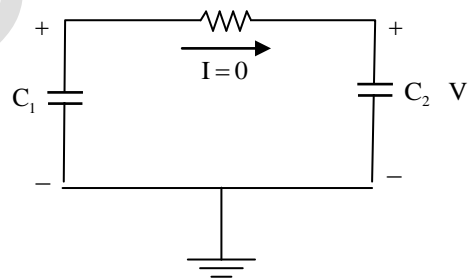
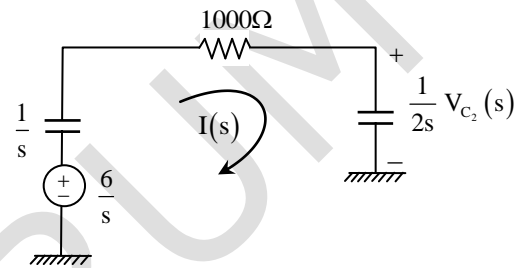
$$q_2 = C_2 V_2 = 2 \times 2 = 4C$$

**Method-2:** (Charge conservation) at  $t = \infty$

$$C_1 V_1 (\text{initial charge}) = (C_1 + C_2) V_{\text{net}}$$

$$\Rightarrow 6 = 3V \Rightarrow V = 2 = V_2 = V_1$$

$$\text{Hence } q_2 = 2 \times 2 = 4C$$



21. A pitot-static tube is used to estimate the velocity of an incompressible fluid of density  $1\text{kg} / \text{m}^3$ . If the pressure difference measured by the tube is  $200\text{N} / \text{m}^2$ , the velocity of the fluid, assuming the pitot-tube coefficient to be 1.0, is \_\_\_\_\_ m/s.

**Key: (20)**

**Sol:** For a Pitot-static tube it is given that

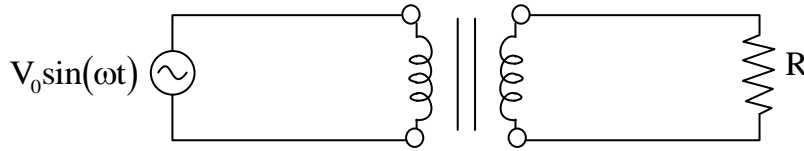
$$\text{density } (\rho) = 1\text{kg} / \text{m}^3$$

$$\text{Pressure difference } (\Delta P) = 200\text{N} / \text{m}^2$$

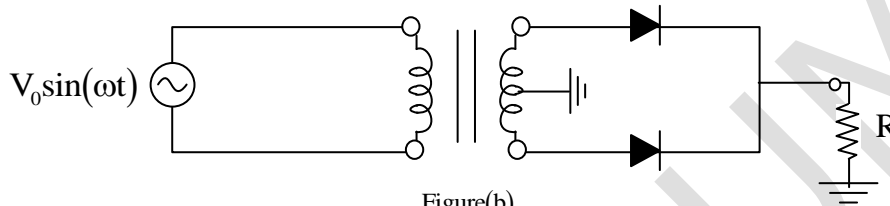
$$\text{Pitot tube co-efficient} = 1$$

$$\text{Velocity, } V = \sqrt{\frac{2\Delta P}{\rho}} = \sqrt{\frac{2 \times 200}{1}} = \sqrt{400} = 20\text{m/sec}$$

22. In the figures (a) and (b) shown below, the transformers are identical and ideal, except that the transformer in figure (b) is centre-tapped. Assuming ideal diodes, the ratio of the root-mean-square (RMS) voltage across the resistor R in figure (a) to that in figure (b) is



Figure(a)



Figure(b)

- (A)  $\sqrt{2} : 1$       (B) 2:1      (C)  $2\sqrt{2} : 1$       (D) 4:1

**Key:** (\*)

23. In a cascade control system, the closed loop transfer function of the inner loop may be assumed to have a single time-constant  $\tau_1$ . Similarly, the closed loop transfer function of the outer loop may be assumed to have a single time-constant  $\tau_2$ . The desired relationship between  $\tau_1$  and  $\tau_2$  in a well-designed control system is

- (A)  $\tau_1$  is much less than  $\tau_2$   
 (B)  $\tau_1$  is equal to  $\tau_2$   
 (C)  $\tau_1$  is much greater than  $\tau_2$   
 (D)  $\tau_1$  is independent less than  $\tau_2$

**Key:** (A)

**Sol:** For the given cascade control system

$\tau_1$  : Represent time constant of inner loop

$\tau_2$  : Represent time constant of outer loop

We know in a cascade control system  $\tau_1 \ll \tau_2$

24. The loop-gain function  $L(s)$  of a control system with unity feedback is given to be

$$L(s) = \frac{k}{(s+1)(s+2)(s+3)}, \text{ where } k > 0. \text{ If the gain cross-over frequency of the loop-gain}$$

function is less than its phase cross-over frequency, the closed-loop system is

- (A) unstable (B) marginally stable  
(C) conditionally stable (D) stable

**Key: (D)**

**Sol:** We know in general when  $\omega_{pc} > \omega_{gc}$  the control system is stable.

25. Thermocouples measure temperature based on

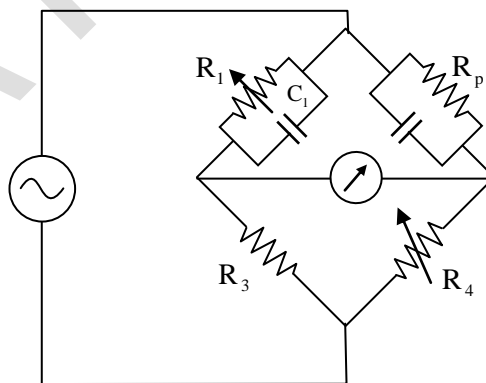
- (A) Photoelectric effect (B) Seebeck effect  
(C) Hall effect (D) Thermal expansion

**Key: (B)**

**Sol:** Thermocouple operating principle is a temperature difference across a metal produces emf which is known as Seebeck effect.

**Q. No. 26 to 55 Carry Two Marks Each**

26. The parallel resistance-capacitance bridge shown below has a standard capacitance value of  $C_1 = 0.1\mu\text{F}$  and a resistance value of  $R_3 = 10\text{k}\Omega$ . The bridge is balanced at a supply frequency of 100Hz for  $R_1 = 375\text{k}\Omega$ ,  $R_3 = 10\text{k}\Omega$  and  $R_4 = 14.7\text{ k}\Omega$ .



The value of the dissipation factor  $D = 1/(\omega R_p C_p)$  of the parallel combination of  $C_p$  and  $R_p$  is \_\_\_\_\_. (Rounded off to three decimal places).

**Key: (0.042)**

**Sol:** Given that  $C_1 = 0.1\mu\text{f}$ ,  $R_3 = 10\text{k}\Omega$ ,  $R_1 = 375\text{k}\Omega$ ,  $R_3 = 10\text{k}\Omega$

$$R_4 = 14.7\text{k}\Omega, \quad D = \frac{1}{\omega R_p C_p}, \quad f_s = 100\text{Hz}.$$

At balance condition

$$\bar{Z}_1 \bar{Z}_4 = \bar{Z}_3 \bar{Z}_p$$

$$\Rightarrow \frac{R_1 R_4 \left( \frac{-j}{\omega C_1} \right)}{R_1 - \frac{j}{\omega C_1}} = \frac{R_3 \left\{ R_p \left( \frac{-j}{\omega C_p} \right) \right\}}{R_p - \frac{j}{\omega C_p}}$$

$$\Rightarrow \frac{-j R_1 R_4}{R_1 \omega C_1 - j} = \frac{-j R_3 R_p}{R_p \omega C_p - j}$$

$$\Rightarrow R_1 R_4 (R_p \omega C_p - j) = R_3 R_p (R_1 \omega C_1 - j)$$

$$\Rightarrow R_1 R_4 R_p \omega C_p - j R_1 R_4 = R_3 R_p R_1 \omega C_1 - j R_3 R_p$$

(Imaginary part and real part will be same at both sides from balance condition).

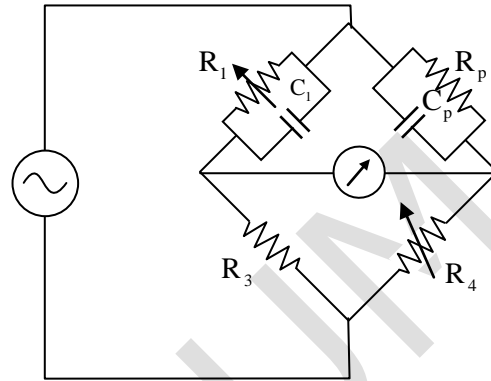
$$R_1 R_4 = R_3 R_p \quad \dots(1)$$

$$R_p = \frac{R_1 R_4}{R_3} = \frac{375 \times 14.7}{10} = 551.25\text{k}\Omega$$

$$R_1 R_4 R_p \omega C_p = R_3 R_p R_1 \omega C_1$$

$$C_p = \frac{R_3 C_1}{R_4} = \frac{10 \times 10^3 \times 0.1 \times 10^{-6}}{14.7 \times 10^3} = 0.068\mu\text{F}$$

$$\text{Dissipation factor, } D = \frac{1}{(2\pi f_s R_p C_p)} = \frac{1}{2\pi \times 100 \times 551.25 \times 10^3 \times 0.068 \times 10^{-6}} = 0.042$$



27. The curve  $y = f(x)$  is such that the tangent to the curve at every point  $(x, y)$  has a Y-axis intercept  $c$ , given by  $c = -y$ . Then,  $f(x)$  is proportional to

- (A)  $x^{-1}$                       (B)  $x^2$                       (C)  $x^3$                       (D)  $x^4$

**Key: (B)**

**Sol:** Equation of the tangent to the curve  $y = f(x)$  at a point  $\Delta_{xy} P(x_1, y_1)$  is  $y - y_1 = m(x - x_1)$ ,

$$\text{where } m = \left( \frac{dy}{dx} \right)_p \text{ or } f'(x)$$

Using y- intercept, we get  $y_1 - mx_1 = -y_1$

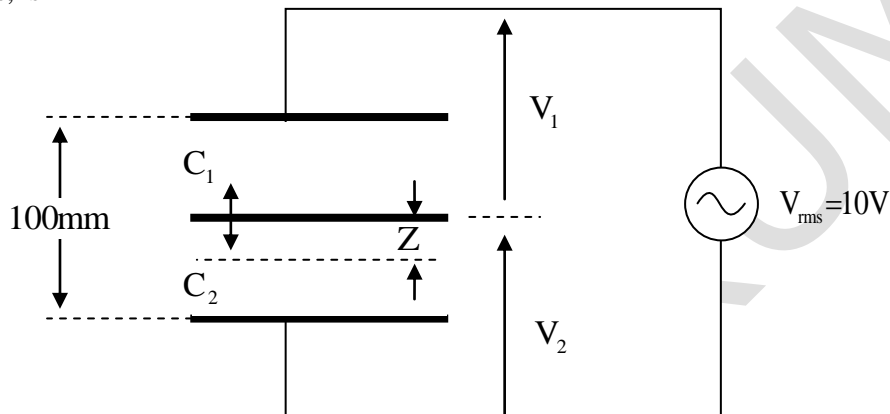
$$\Rightarrow m = \frac{2y_1}{x_1} \Rightarrow m = \frac{2y}{x} \text{ (dropping suffix)}$$



$$\Rightarrow \frac{dy}{dx} = \frac{2y}{x} \Rightarrow \frac{dy}{y} = 2 \frac{dx}{x}$$

Integrating, we get  $y = k \cdot x^2 \Rightarrow f(x)$  is proportional to  $x^2$

28. A differential capacitive sensor with a distance between the extreme plates 100mm is shown in figure below. The difference voltage  $\Delta V = V_1 - V_2$ , where  $V_1$  and  $V_2$  are the rms values, for a downward displacement of 10mm of the intermediate plate from the central position, in volts, is



(A) 0.9

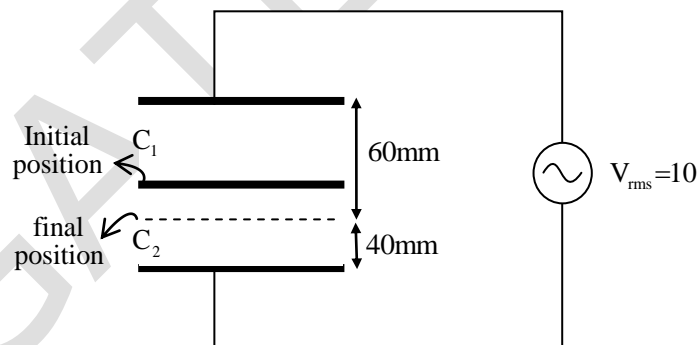
(B) 1.0

(C) 1.1

(D) 2

**Key: (D)**

**Sol:** From the given figure



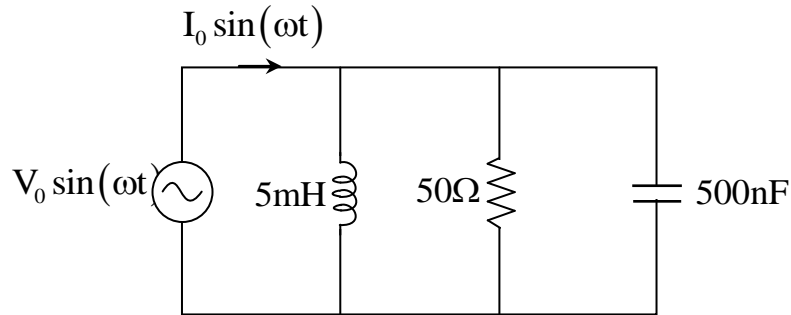
$$C_1 = \frac{\epsilon A}{60\text{mm}}; \quad C_2 = \frac{\epsilon A}{40\text{mm}}$$

$$\Rightarrow V_1 = \frac{V_{\text{rms}} C_2}{C_1 + C_2} = \frac{10 \times \frac{\epsilon A}{40\text{mm}}}{\frac{\epsilon A}{60\text{mm}} + \frac{\epsilon A}{40\text{mm}}} = \frac{10 \times \frac{1}{40}}{\frac{1}{60} + \frac{1}{40}} = 6\text{V}$$

$$\Rightarrow V_2 = V_s - V_1 = 10 - 6 = 4$$

$$\Rightarrow \Delta V = V_1 - V_2 = 6 - 4 = 2\text{V}.$$

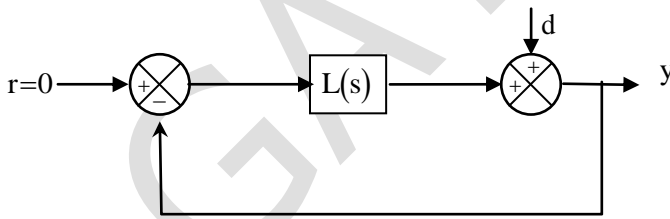
29. In the circuit shown below, the angular frequency  $\omega$  at which the current is in phase with the voltage is \_\_\_\_\_ rad/s.



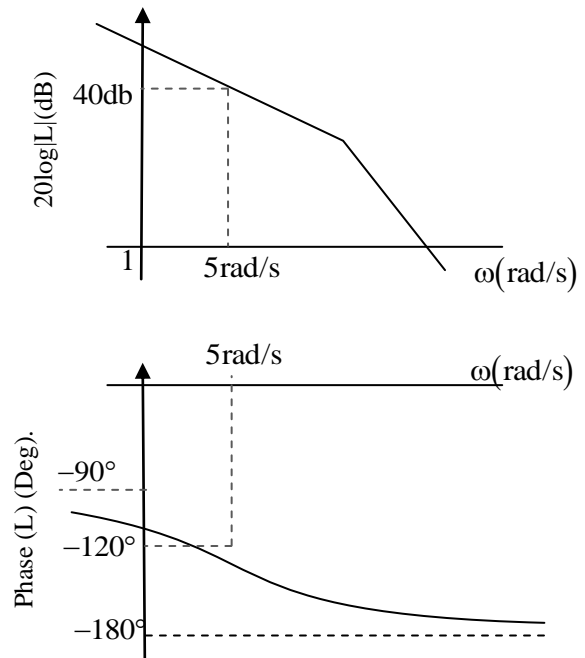
**Key:** (20,000)

**Sol:** → For a parallel RLC circuit, the resonant frequency is  $\omega_0 = \frac{1}{\sqrt{LC}}$   
 $= \frac{1}{\sqrt{5 \times 10^{-3} \times 500 \times 10^{-9}}} = 20,000 \text{ rad/sec.}$

30. The forward path transfer function  $L(s)$  of the control system shown in figure (a) has the asymptotic Bode plot shown in Figure (b). If the disturbance  $d(t)$  is given by  $d(t) = 0.1 \sin(\omega t)$  where  $\omega = 5 \text{ rad/s}$ , the steady-state amplitude of the output  $y(t)$  is



Figure(a)



Figure(b)

- (A)  $1.00 \times 10^{-3}$       (B)  $2.50 \times 10^{-3}$       (C)  $5.00 \times 10^{-3}$       (D)  $10.00 \times 10^{-3}$

**Key: (A)**

**Sol:** At  $w = 5$ , gain = 40db

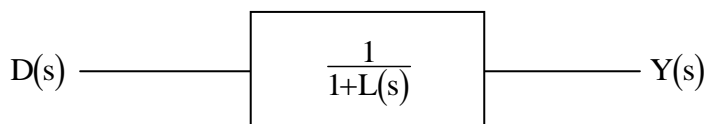
$$20 \log m = 40$$

$$\log m = 2$$

$$m = 10^2 = 100$$

$$\rightarrow \frac{Y(s)}{D(s)} = \frac{1}{1+L(s)} \text{ (By using SFG on the block diagram)}$$

By considering  $y(t)$  as output and  $d(t)$  as input we can redraw the block diagram as



$$\rightarrow |L(s)|_{w=5} = 40\text{db} = 100$$

$$\rightarrow \text{if } d(t) = 0.1 \sin 5t$$

$$\text{then } y(t) = \left[ \frac{1}{1+L(s)} \right]_{w=5} (0.1 \sin 5t)$$

$$= \frac{1}{1+100} (0.1) \sin 5t = (1 \times 10^{-3}) \sin 5t$$

So the steady state amplifier is  $1 \times 10^{-3}$ .

**31.** The function  $p(x)$  is given by  $p(x) = A/x^\mu$  where  $A$  and  $\mu$  are constant with  $\mu > 1$  and  $1 \leq x < \infty$  and  $p(x) = 0$  for  $-\infty < x < 1$ . For  $p(x)$  to be probability density function, the value of  $A$  should be equal to

(A)  $\mu - 1$

(B)  $\mu + 1$

(C)  $1/(\mu - 1)$

(D)  $1/(\mu + 1)$

**Key: (A)**

**Sol:** Since  $P(x)$  is p.d.f  $\Rightarrow \int_{-\infty}^{\infty} p(x) dx = 1$

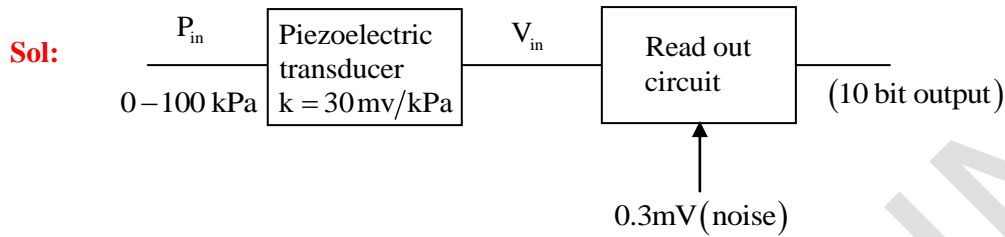
$$\Rightarrow \int_0^1 (0) dx + \int_1^{\infty} \frac{A}{x^\mu} dx = 1$$

$$\Rightarrow A \int_1^{\infty} x^{-\mu} dx = 1 \Rightarrow A \left[ \frac{1}{(1-\mu) \cdot x^{\mu-1}} \right]_1^{\infty} = 1$$

$$\Rightarrow \frac{A}{1-\mu} [0 - 1] = 1 \Rightarrow A = \mu - 1 \left( \because \mu - 1 > 0 \Rightarrow \frac{1}{x^{\mu-1}} \rightarrow 0 \text{ when } x \rightarrow \infty \right)$$

32. A piezoelectric transducer with sensitivity of 30mV/kPa is intended to be used in the range of 0 kPa to 100 kPa. The readout circuit has a peak noise amplitude of 0.3mV and measured signals over the full pressure range are encoded with 10 bits. The smallest pressure that produces a non-zero output, in units of Pa, is approximately
- (A) 10                      (B) 100                      (C) 240                      (D) 300

**Key: (B)**



The read out circuit shown in the above figure is nothing but an ADC. Since we are expecting the a non zero output, then it input should must be at least

$$V_{in} \geq [\text{Resolution of read out}] + [\text{noise}]$$

$$\text{Resolution of read out} = \frac{[V_{in}]_{\max}}{2^{10}} = \frac{100\text{kPa} \times 30 \frac{\text{mV}}{\text{kPa}}}{2^{10}} = \frac{300}{1024} = 2.92\text{mV}$$

$$\Rightarrow V_{in} \geq 2.92 + 0.3$$

$$\Rightarrow V_{in} \geq 3.22\text{mV}$$

$$V_{in} = k \times P_{in}$$

$$P_{in} = \frac{V_{in}}{k} = \frac{3.22\text{mV}}{30 \text{ mV/kPa}} = 0.1076 \text{ kPa} = 107 \text{ Pa} \approx 100\text{Pa}$$

33. A 100W light source emits uniformly in all directions. A photo detector having a circular active area whose diameter is 2cm is placed 1m away from the source, normal to the incident light. If the responsivity of the photo detector is 0.4A/W, the photo-current generated in the detector, in units of mA, is \_\_\_\_
- (A) 1                      (B) 4                      (C) 100                      (D) 400

**Key: (Insufficient information)**

34.  $X = X_1X_0$  and  $Y = Y_1Y_0$  are 2- bit binary numbers. The Boolean function S that satisfies the condition "If  $X > Y$ , then  $S=1$ ", in its minimized form, is

(A)  $X_1Y_1 + X_0Y_0$

(B)  $X_1\bar{Y}_1 + X_0\bar{Y}_0\bar{Y}_1 + X_0\bar{Y}_0X_1$

(C)  $X_1\bar{Y}_1 + X_0\bar{Y}_0$

(D)  $X_1Y_1 + X_0\bar{Y}_0Y_1 + X_0\bar{Y}_0\bar{X}_1$

**Key: (B)**

**Sol:**  $\left. \begin{matrix} X_1 & X_0 \\ Y_1 & Y_0 \end{matrix} \right\}$  when we compare X and Y, then  $X > Y$  can happen in 2 possible waves

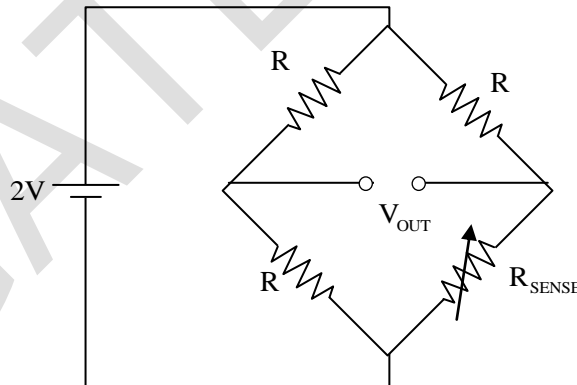
(i)  $X_1 = 1, Y_1 = 0 \Rightarrow X_1 \bar{Y}_1$

(ii)  $(X_1 = Y_1) \text{ and } (X_0 = 1, Y_0 = 0) \Rightarrow (X_1 \odot Y_1)(X_0 \bar{Y}_0)$

So the expression for  $X > Y$  will be

$$\begin{aligned} S &= X_1 \bar{Y}_1 + [(X_1 \odot Y_1) \cdot (X_0 \bar{Y}_0)] \\ &= X_1 \bar{Y}_1 + [(X_1 Y_1 + \bar{X}_1 \bar{Y}_1)(X_0 \bar{Y}_0)] \\ &= (X_1 \bar{Y}_1 + X_1 Y_1 X_0 \bar{Y}_0) + \bar{X}_1 \bar{Y}_1 X_0 \bar{Y}_0 \\ &= [X_1 (\bar{Y}_1 + Y_1 X_0 \bar{Y}_0)] + \bar{X}_1 \bar{Y}_1 X_0 \bar{Y}_0 \\ &= [X_1 (\bar{Y}_1 + X_1 X_0 \bar{Y}_0)] + \bar{X}_1 \bar{Y}_1 X_0 \bar{Y}_0 (\because \bar{A} + AB = \bar{A} + B) \\ &= \underbrace{X_1 \bar{Y}_1 + X_1 X_0 \bar{Y}_0 + \bar{X}_1 \bar{Y}_1 X_0 \bar{Y}_0} \\ &= [\bar{Y}_1 (X_1 + \bar{X}_1 X_0 \bar{Y}_0)] + X_1 X_0 \bar{Y}_0 = \bar{Y}_1 (X_1 + X_0 \bar{Y}_0) + X_1 X_0 \bar{Y}_0 \\ &= X_1 \bar{Y}_1 + X_0 \bar{Y}_0 \bar{Y}_1 + X_0 \bar{Y}_0 X_1 \end{aligned}$$

35. Four identical resistive strain gauges with gauge factor of 2.0 are used in a Wheatstone bridge as shown in the figure below.



Only one of the strain gauges  $R_{\text{SENSE}}$  changes its resistance due to strain. If the output voltage  $V_{\text{OUT}}$  is measure to be 1 mV, the magnitude of strain, in units of micro strain, is

- (A) 1                      (B) 10                      (C) 100                      (D) 1000

**Key: (D)**

**Sol:** The given circuits is Standard Quarter Bridge, whose output voltage is given by

$$V_0 = \frac{V_s}{4} (G.F) \cdot \epsilon$$

Where,

$V_0$  – Output voltage = 1 mV( given)

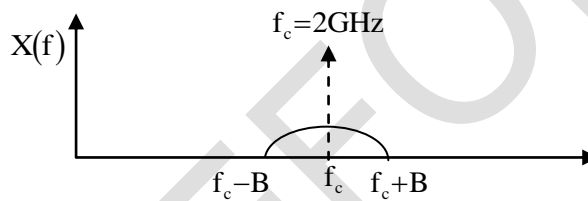
$V_s$  – Supply voltage = 2V (given)

G.F– Gauge Factor = 2 (given)

$\epsilon$  – Strain

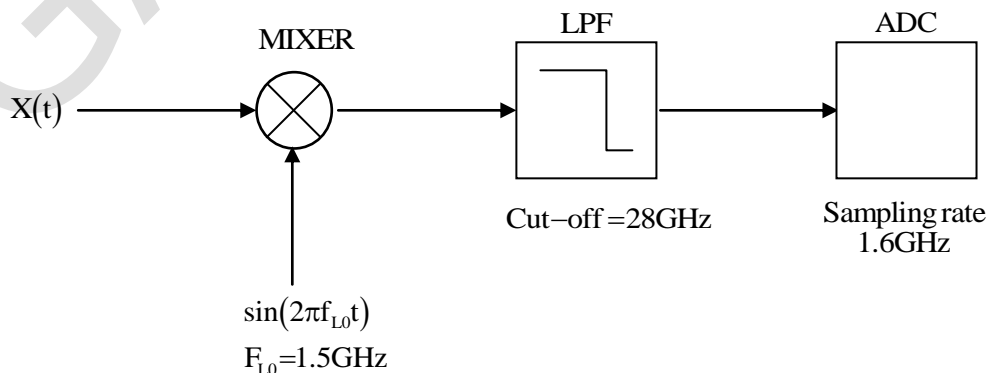
$$\begin{aligned} \Rightarrow \epsilon &= \frac{4V_0}{V_s \cdot (G.F)} = \frac{(4) \times (1 \times 10^{-3})}{(2) \times (2)} = 1 \times 10^{-3} = 10^3 \times 10^{-3} \times 10^{-3} \\ &= 1000 \times 10^{-6} = 1000 \text{ unit of micro strain} \end{aligned}$$

36. A signal  $x(t)$  has a bandwidth  $2B$  about a carrier frequency of  $f_c = 2\text{GHz}$  as shown in figure (a) below.



Figure(a)

In order to demodulate this signal, it is first mixed (multiplied) with a local oscillator of frequency  $f_{LO} = 1.5\text{GHz}$ , and then passed through an ideal low-pass filter (LPF) with a cut-off frequency of  $2.8\text{GHz}$ . The output of the LPF is sent to a digitizer ADC with a sampling rate of  $1.6\text{GHz}$  as shown in figure (b) below.

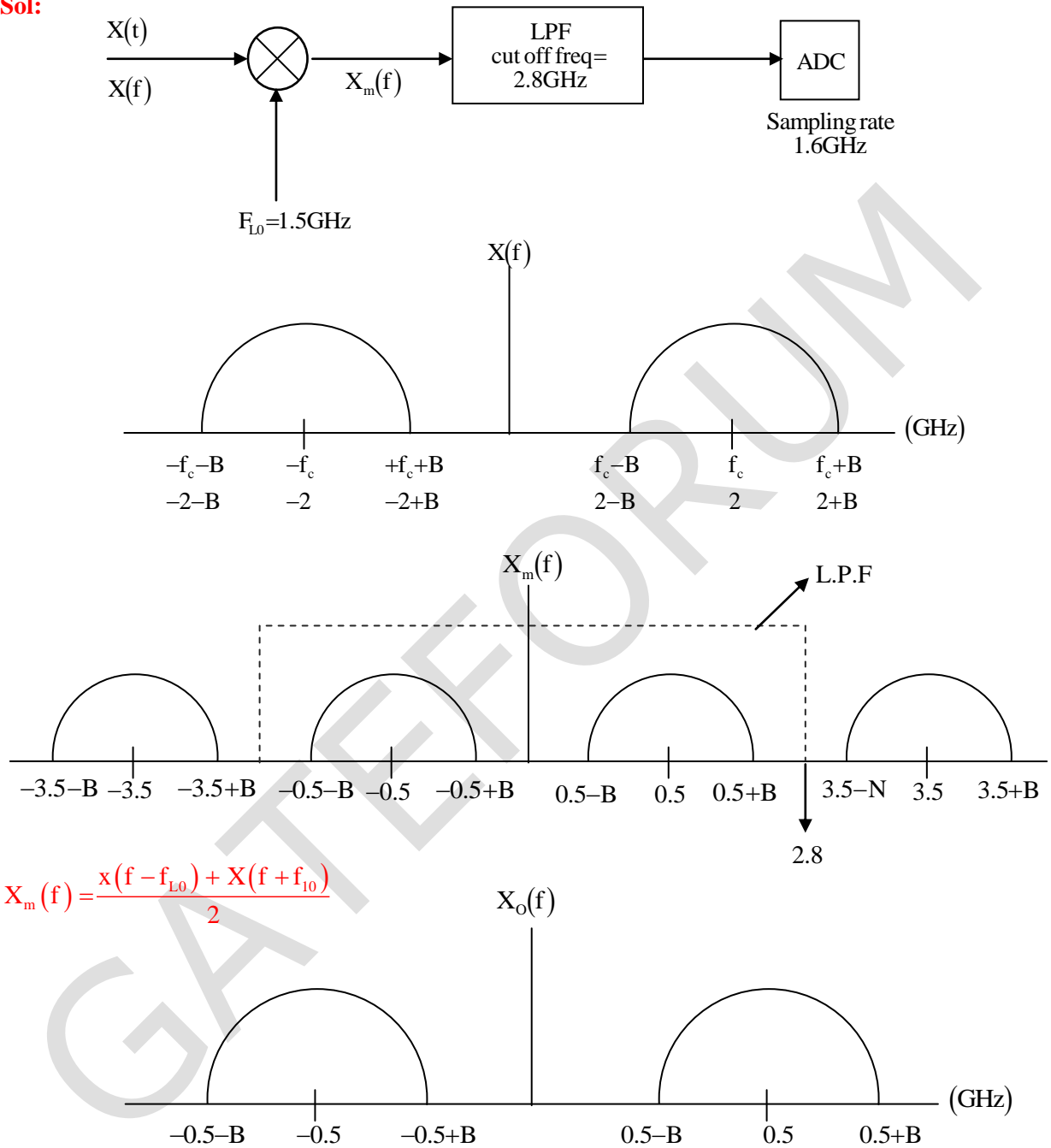


Figure(b)

The maximum value of B so that the signal  $x(t)$  can be reconstructed from its sample according to the Nyquist sampling theorem is \_\_\_\_\_ MHz.

**Key: (300)**

**Sol:**



$$X_m(f) = \frac{x(f - f_{L0}) + X(f + f_{L0})}{2}$$

Nyquist rate of  $X_o(f) = 2(0.5 + B) = 1 + 2B$

The sampling frequency of  $X_o(f)$  should be 1.6 GHz

$$\Rightarrow 1 + 2B = 1.6$$

$$\Rightarrow 2B = 0.6$$

$$\Rightarrow B = 0.3 \text{ GHz} = 300 \text{ MHz}$$

37. A resistance-meter has five measurement range-settings between  $200\Omega$  and  $2\text{ M}\Omega$  in multiplies of 10. The meter measures resistance of a device by measuring a full-range voltage of  $2\text{ V}$  across the device by passing an appropriate constant current for each range-setting. If a device having a resistance value in the range  $8\text{ k}\Omega$  to  $12\text{ k}\Omega$  and a maximum power rating of  $100\mu\text{ W}$  is to be measured safely with this meter, the choice for range-setting on the meter for best resolution in measurement, in  $\text{k}\Omega$  is
- (A) 2                                      (B) 20                                      (C) 200                                      (D) 2000

**Key:** (C)

**Sol:**  $8\text{ k}\Omega < R < 12\text{ k}\Omega$

→ When the resistance is  $20\text{ k}\Omega$

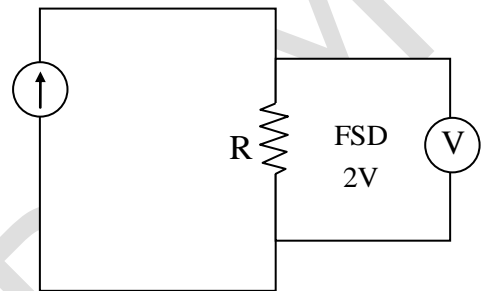
$$I = \frac{2\text{ V}}{20\text{ k}\Omega} = \frac{1}{10} \text{ mA}$$

$$P_{\text{max}} = I^2 R_{\text{max}} = \left(\frac{1}{10} \times 10^{-3}\right)^2 (12 \times 10^3) = 120\mu\text{ W} > 100\mu\text{ W} \text{ (so not suitable)}$$

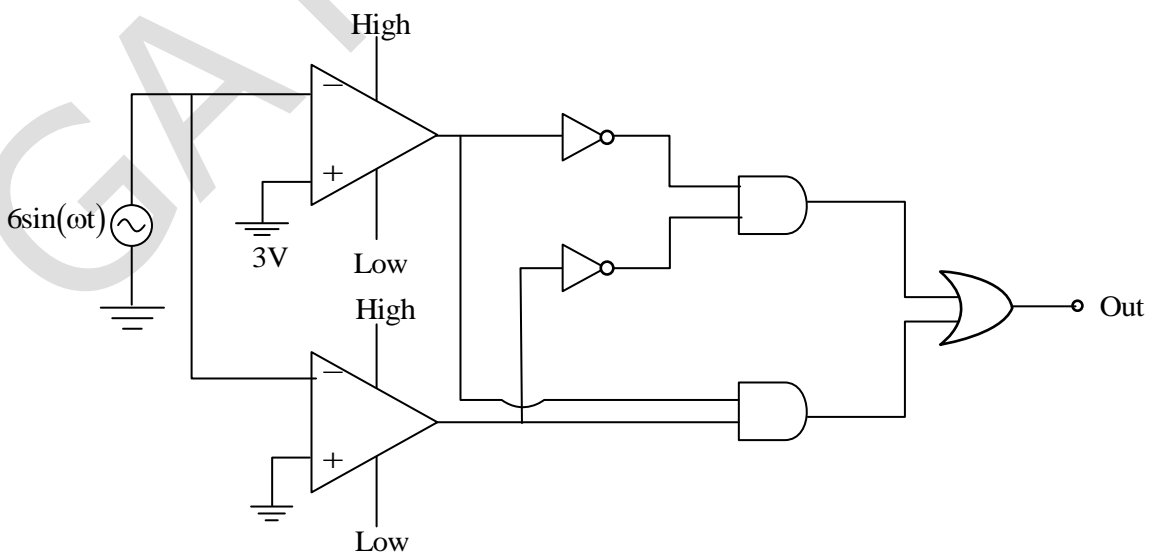
→ When the resistance is  $200\text{ k}\Omega$

$$I = \frac{2\text{ V}}{200\text{ k}\Omega} = \frac{1}{100} \text{ mA}$$

$$P_{\text{max}} = I^2 R_{\text{max}} = \left(\frac{1}{100} \times 10^{-3}\right)^2 \times (12 \times 10^3) = 1.2\mu\text{ W}$$



38. In the circuit shown below, assume that the comparators are ideal and all components have zero propagation delay. In one period of the input signal  $V_{\text{in}} = 6\sin(\omega t)$ , the fraction of the time for which the output OUT is in logic state HIGH is

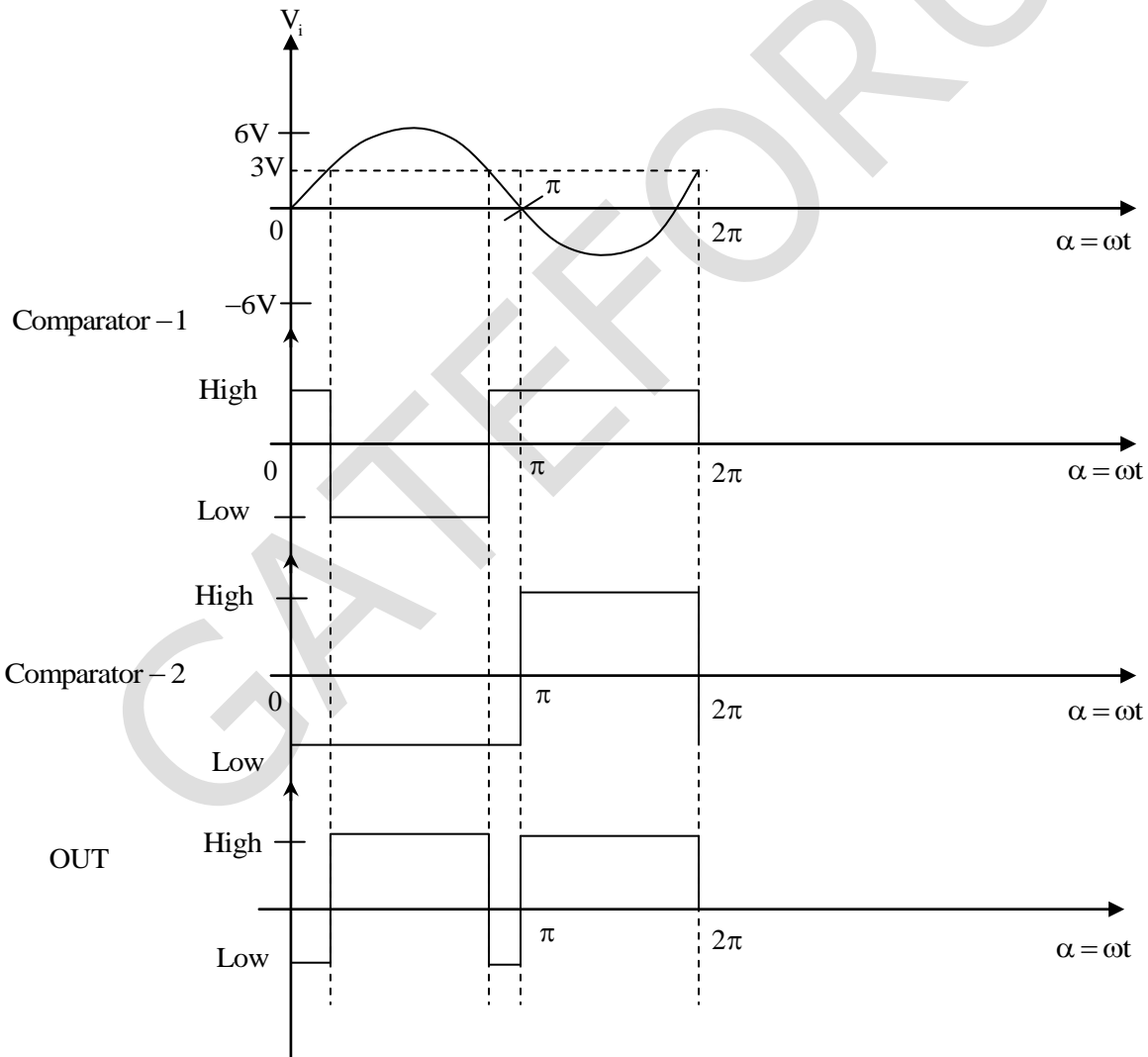
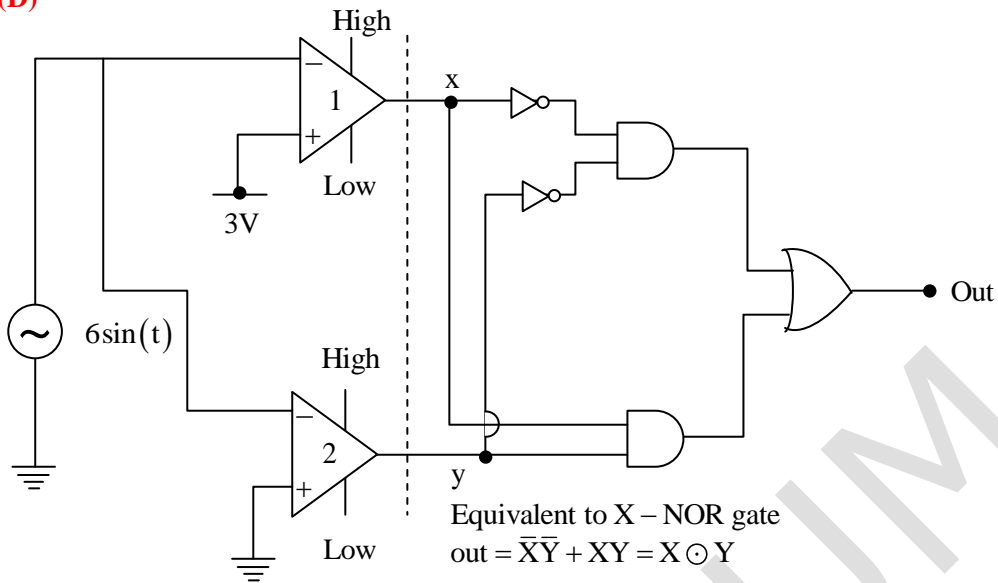


- (A) 1/12                                      (B) 1/2                                      (C) 2/3                                      (D) 5/6



**Key: (D)**

**Sol:**



Operator of comparator:

$$V_+ > V_- \rightarrow \text{output is logic 1}$$

$$V_+ < V_- \rightarrow \text{output is logic } < 0$$

For comparator 1:

$$0 < \alpha < 2\pi$$

When input is less than  $3V$ ,

$$V_+ - V_- > 0 \text{ so output } x \rightarrow \text{High}$$

When input is more than  $3V$

$$V_+ - V_- < 0 \text{ So output } x \rightarrow \text{low}$$

For comparator 2:

When:  $0 < \alpha < \pi$

$$V_+ - V_- < 0 \text{ so output } Y \rightarrow \text{low}$$

When:  $\pi < \alpha < 2\pi$

$$V_+ - V_- > 0, \text{ so output } Y = \text{High}$$

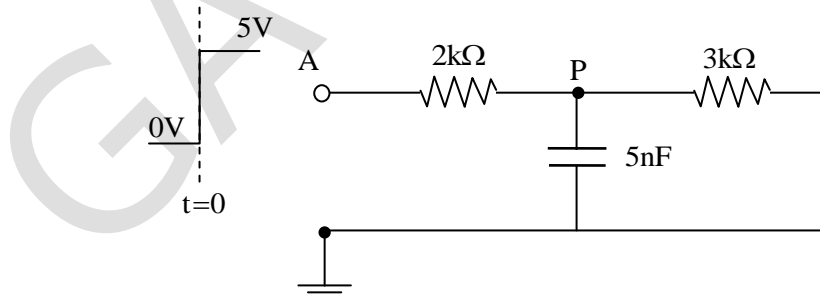
$$\text{Now, } 3 = 6 \sin(\omega t) \Rightarrow \theta = (\omega t) = \sin^{-1}\left(\frac{3}{6}\right) = \frac{\pi}{6}$$

$$T = \frac{T_{\text{ON}}}{T} = \frac{\pi - 2\theta + \pi}{2\pi} = \frac{2\pi - \frac{\pi}{3}}{2\pi} = \frac{5}{6}$$

Therefore, fraction of time of which XNOR (out) is high:

$$T = \frac{5}{6}$$

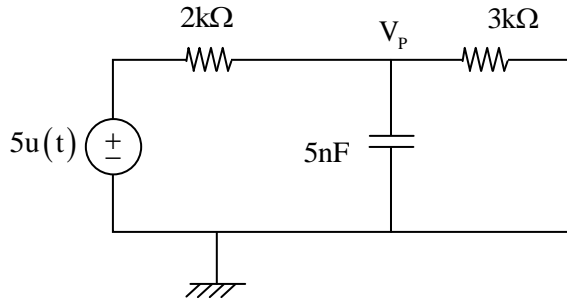
39. In the circuit shown below, a step input voltage of magnitude  $5V$  is applied at node A at time  $t=0$ .



If the capacitor has no charge for  $t \leq 0$ , the voltage at node P at  $t=6\mu s$  is \_\_\_\_\_ V.  
(Rounded off to two decimal places)

**Key: (1.89)**

**Sol:** The given circuit is

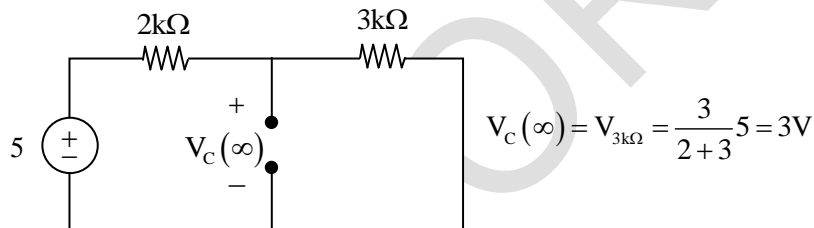


Voltage at node P = Voltage of capacitor

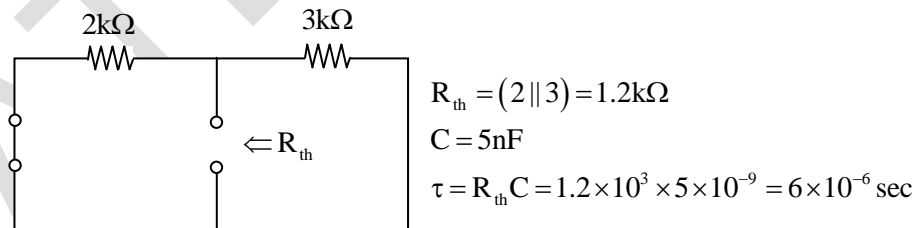
For 1<sup>st</sup> order RC N/W, the voltage across capacitor is given by

$$V_c(t) = V_c(\infty) [1 - e^{-t/\tau}] \quad [\because V_c(0^-) = 0 \text{ given}] \quad \text{Vlc}$$

At  $t = \infty$ , N/W is in steady state and the capacitor is open circuit.



To calculate  $\tau = R_{th}C$ , the independent voltage source is short circuited, & Rth is obtained from capacitor terminal.



$$R_{th} = (2/3) = 1.2k\Omega$$

$$C = 5nF$$

$$\tau = R_{th}C = 1.2 \times 10^3 \times 5 \times 10^{-9} = 6 \times 10^{-6} \text{ sec}$$

$$V_c(t) = V_p = V_c(\infty) (1 - e^{-t/\tau}) = 3(1 - e^{-t/6 \times 10^{-6}})$$

$$V_c(6\mu\text{sec}) = 3 \left( 1 - e^{-\frac{6 \times 10^{-6}}{6 \times 10^{-6}}} \right) = 3(1 - e^{-1}) = 1.89V$$

40. The frequency response of a digital filter  $H(\omega)$  has the following characteristics

Passband:  $0.95 \leq |H(\omega)| \leq 1.05$  for  $0 \leq \omega \leq 0.3\pi$  and

Stopband:  $0 \leq |H(\omega)| \leq 0.005$  for  $0.4\pi \leq \omega \leq \pi$ ,

Where  $\omega$  is the normalized angular frequency in rad/sample. If the analog upper cut off frequency for the passband of the above digital filter is to be 1.2 kHz, then the sampling frequency should be \_\_\_\_\_ kHz.

**Key: (8)**

**Sol:** For the given digital filter  $H(\omega)$ , having

Passband:  $0.95 \leq |H(\omega)| \leq 1.05$ ;  $0 \leq \omega \leq 0.3\pi$

Stop band:  $0 \leq |H(\omega)| \leq 0.005$ ;  $0.4\pi \leq \omega \leq \pi$

$\omega$ : Represent normalized angular frequency (rad/sample) which is nothing but the digital frequency.

It is clear that is a low pass filter (practical), with digital cut-off frequency  $(\omega_d) = 0.3\pi$  and its corresponding analog cut-off frequency  $(f_a) = 1.2$  kHz

The relation between analog frequency  $(\omega_a)$  and the digital frequency  $(\omega_d)$  is given by

Let  $x(a) = \cos \omega_a t$

$$x(nT_s) = \cos \omega_a T_s = \cos n \frac{\omega_a}{f_s} = \cos n \omega_d$$

$$\omega_d = \frac{\omega_a}{f_s}$$

$\omega_d$  : digital frequency

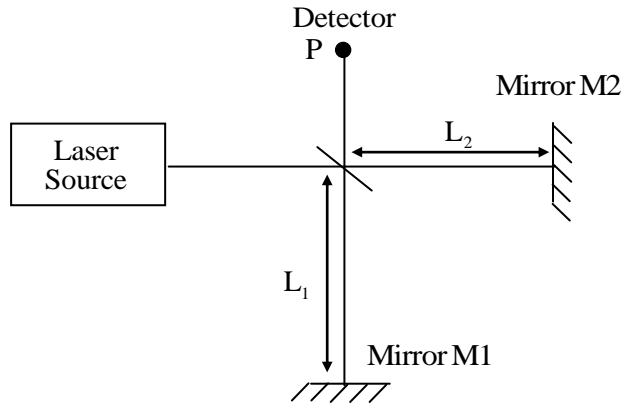
$\omega_a$  : Analog frequency

$f_s$  : sampling frequency

$$\omega_d = \frac{\omega_a}{f_s}$$

$$f_s = \frac{\omega_a}{\omega_d} = \frac{2\pi f_a}{0.3\pi} = \frac{2\pi \times 1.2 \times 10^3}{0.3\pi} = 8 \times 10^3 = 8 \text{ kHz}$$

41. Consider a Michelson interferometer as shown in the figure below. When the wavelength of the laser light source is switched from 400 nanometer to 500 nanometer, it is observed that the intensity measured at the output P goes from a minimum to a maximum.



This observation is possible when the smallest path difference between the two arms of the interferometer is \_\_\_\_\_ nanometer.

**Key: (250)**

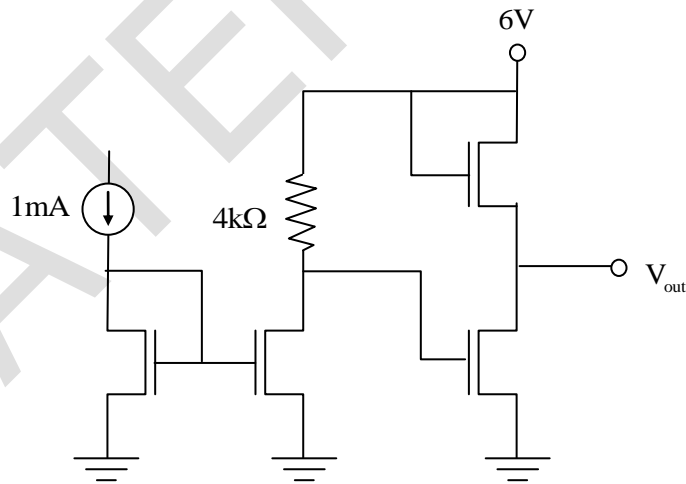
**Sol:** In Michelson Interferometer

$$n\lambda = 2d \text{ (where } d \text{ is path difference)}$$

→ When the wavelength of light source is adjusted to 500nm then

$$d = \frac{\lambda}{2} = \frac{500}{2} = 250\text{nm}$$

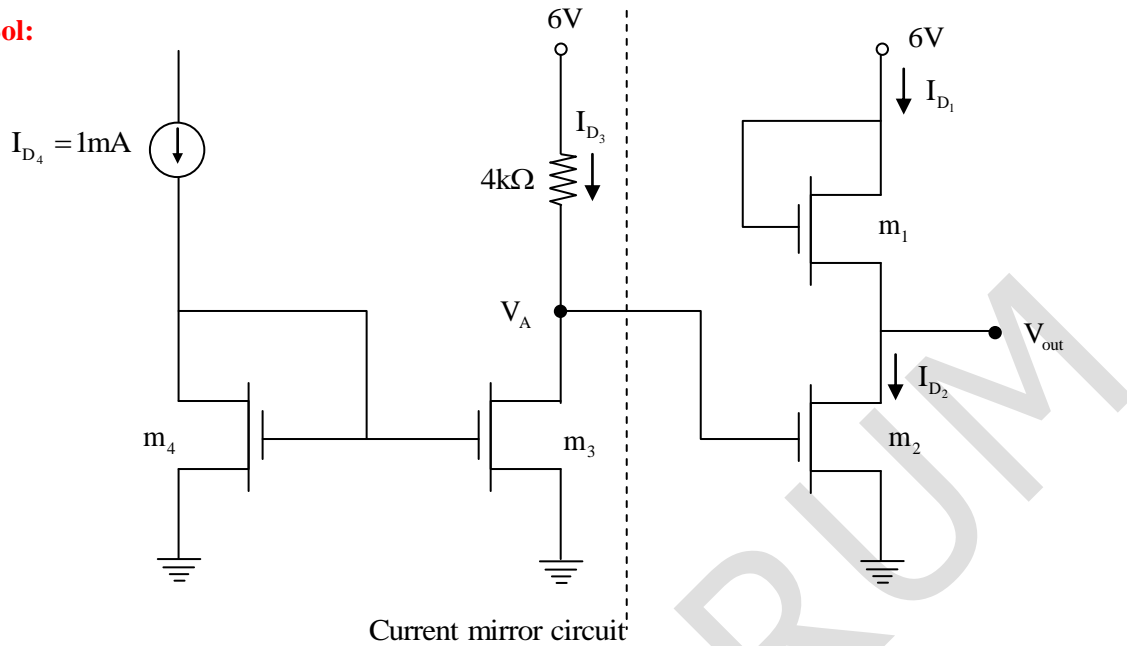
42. In the circuit shown below, all transistors are n-channel enhancement mode MOSFETs. They are identical and are biased to operate in saturation mode.



Ignoring channel length modulation, the output voltage  $V_{out}$  is \_\_\_\_\_ V.

**Key: (4)**

**Sol:**



First stage of the circuit is current mirror circuit:

$$V_{GS3} = V_{GS4} \text{ so that } I_{D3} = I_{D4} = 1 \text{ mA}$$

$$V_A = 6V - 4k \times 1m = 2V$$

For 2<sup>nd</sup> stage of the circuit as per the question all the transistor working into saturation

$$\text{So, } I_{D1} = I_{D2}$$

$$(V_{GS1} - V_t)^2 = (V_{GS2} - V_t)^2$$

$$V_{GS1} - V_t = \pm(V_{GS2} - V_t)$$

As threshold voltage ( $v_t$ ) unit mentioned in the question, so take only +ve sign.

$$V_{GS1} - V_t = V_{GS2} - V_t$$

$$V_{G1} - V_{s1} = V_{G2} - V_{s2}$$

$$6V - V_{s1} = 2V - 0$$

$$\therefore V_{s1} = 4V$$

$$V_{s1} = V_{D2} = V_{out} = 4V$$

43. In a control system with unity gain feedback, the transfer function of the loop-gain function is  $L(s) = 9e^{-0.1s} / s$ . The phase margin of the loop-gain function  $L(S)$  is \_\_\_\_\_ degree.

**Key:** (38.44)

**Sol:** Given,

The transfer function of the loop-given function is  $L(s) = 9e^{-0.1s} / s$ ,  $H(s) = 1$

To calculate the phase margin, we need to calculate find  $\omega_{gc}$ .

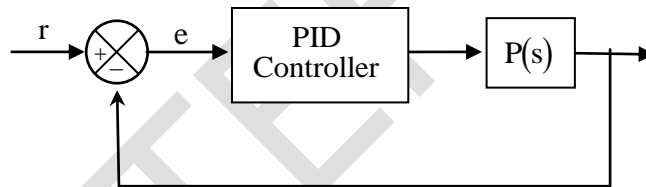
$$|L(j\omega)| = 1 \Rightarrow \left| \frac{9}{\omega} \right| = 1$$

$$\omega_{gc} = 9 \text{ rad/sec}$$

$$\Rightarrow \phi \text{ at } \omega_{gc} = -90^\circ + \angle -0.1 \times 9 \times \frac{180}{\pi} = -90^\circ - 51.56 = -141.56^\circ$$

$$\text{Phase margin} = 180 + \phi_{at\omega_{gc}} = 180 - 141.56 = 38.44^\circ$$

44. In the control system shown in the figure below, a reference signal  $r(t) = t^2$  is applied at time  $t = 0$ . The control system employs a PID controller  $C(s) = K_p + K_i / s + K_d s$  and the plant has a transfer function  $P(s) = 3/s$ . If  $K_p = 10$ ,  $K_i = 1$  and  $K_d = 2$ , the steady state value of  $e$  is



(A) 0

(B) 2/3

(C) 1

(D)  $\infty$

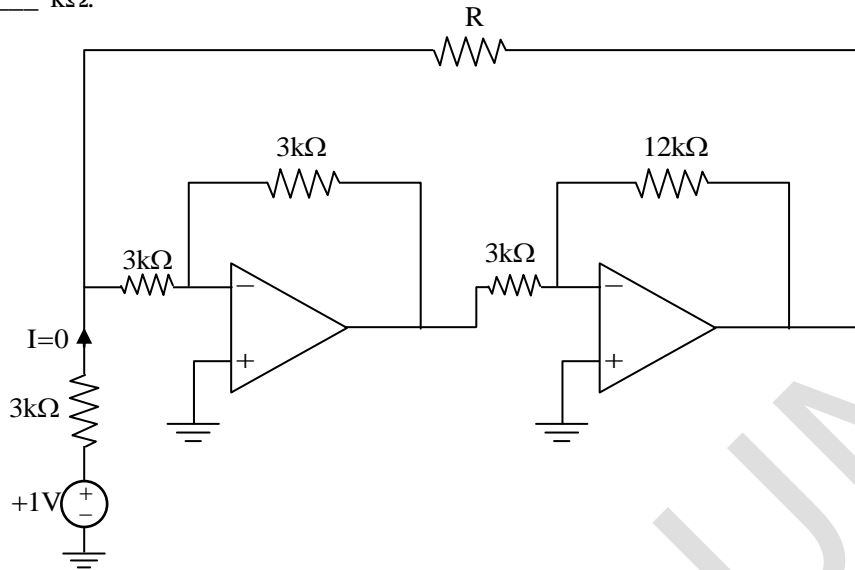
**Key:** (B)

**Sol:** The steady state value  $e = \lim_{s \rightarrow 0} \frac{sR(s)}{1 + G(s)H(s)}$  (Valid for unity feedback)

$$G(s)H(s) = C(s)P(s) = \frac{3}{s} \left\{ 10 + \frac{1}{s} + 2s \right\} = \frac{3}{s} \left\{ \frac{10s + 1 + 2s^2}{s} \right\} = \frac{3}{s^2} (10s + 1 + 2s^2)$$

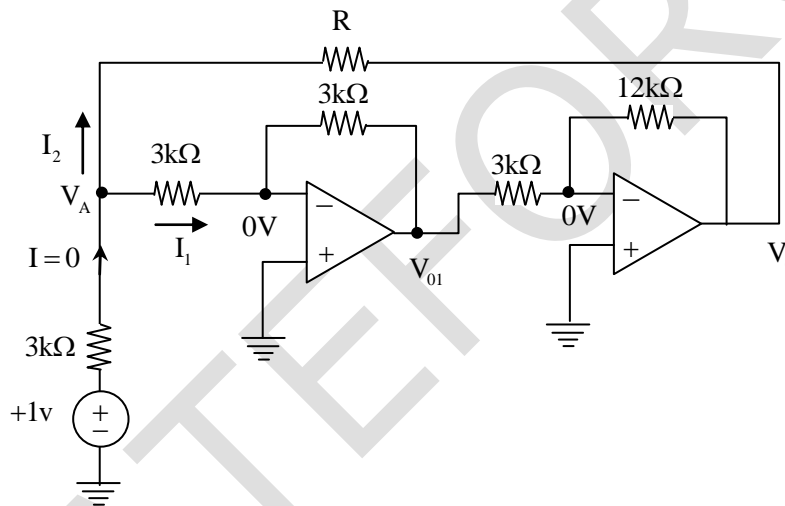
$$\therefore e_{ss} = \lim_{s \rightarrow 0} \frac{s \times \frac{2}{s^3}}{1 + \frac{3}{s^2} (10s + 1 + 2s^2)} = \frac{2}{3}$$

45. In the circuit shown below, all OPAMPS are ideal. The current  $I = 0A$  when the resistance  $R = \underline{\hspace{2cm}}$  k $\Omega$ .



**Key: (9)**

**Sol:**



Apply KCL at node  $V_A$

$$I = I_1 + I_2$$

$$I = \frac{V_A - 0}{3k} + \frac{V_A - V_0}{R} \quad \dots (1)$$

$$\text{From the circuit } V_{01} = \frac{-3k}{3k} \times V_A = -V_A$$

$$\text{And } V_0 = \frac{-12k}{3k} \times V_{01} = -4 \times (-V_A) = +4V_A$$

Substitute the value of  $V_0$  in equation (1) and make  $I = 0$ , then

$$I = \frac{V_A}{3k} + \frac{V_A - 4V_A}{R} \Rightarrow 0 = \frac{V_A}{3k} - \frac{3V_A}{R}$$

$$\frac{3V_A}{R} = \frac{V_A}{3k} \Rightarrow \therefore R = 9 \text{ k}\Omega$$



46. In a microprocessor with a 16bit address bus, the most significant address line A15 to A12 are used to select a 4096 word memory unit, while line A0 to A11 are used to address a particular word in the memory unit. If the 3 least significant lines of the address bus A0 to A2 are short-circuited to ground, the addressable number of words in the memory unit is \_\_\_\_\_.

**Key:** (512)

**Sol:** → From the given information it is clear that  $A_{15}A_{14}A_{13}A_{12}$  are used for chip selection purpose

→  $A_2A_1A_0$  are shorted means logic 0.

→ Out of 16 address line 7 address line have fixed value so effective's 9 address line whose values can be varied, so total number of words that can be accessed is  $2^9 = 512$ .

47. A discrete-time signal  $x[n] = e^{j\left(\frac{5\pi}{3}\right)n} + e^{j\left(\frac{\pi}{4}\right)n}$  is down-sampled to the signal  $x_d[n]$  such that  $x_d[n] = x[4n]$ . The fundamental period of the down-sampled signal  $x_d[n]$  is \_\_\_\_\_.

**Key:** (6)

**Sol:** It is given that  $x(n) = e^{j\left(\frac{5x}{3}\right)n} + e^{j\left(\frac{\pi}{4}\right)n}$

We need to find fundamental period of  $x_d(n) = x(4n)$

$$\Rightarrow x(n) = e^{j\left(\frac{5x}{3}\right)n} + e^{j\left(\frac{x}{4}\right)n}$$

$$\Rightarrow x(4n) = e^{j\left(\frac{5x}{3}\right)(4n)} + e^{j\left(\frac{x}{4}\right)(4n)} = e^{j\left(\frac{20x}{3}\right)n} + e^{j(x)n}$$

For  $e^{j\left(\frac{20x}{3}\right)n}$ , the fundamental period is

$$N_1 = \frac{2x}{\omega_0} k \quad \left( \because \frac{2x}{\omega_0} \text{ is integer in this case} \right)$$

$$= 2x \times \frac{3}{20\pi} \times k = \frac{3}{10} k = 3 \quad (\text{if } k = 10)$$

For  $e^{j(x)n}$

$$N_2 = \frac{2x}{\omega_0} k \quad \left( \because \frac{2x}{\omega_0} \text{ is integer in this case} \right)$$

$$= \frac{2x}{x} k = 2k = 2 \quad (\text{if } k = 1)$$

Since both of individual signals are periodic with period  $N_1$  and  $N_2$ , then their combination is also periodic and is given by

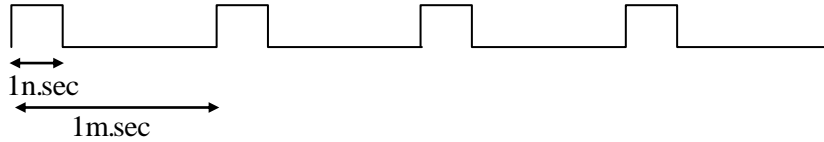
$$N = \text{LCM}(N_1, N_2) = \text{LCM}(3, 2) = 6$$

So the fundamental period of  $x_d(n) = 6$

48. A pulsed laser emits rectangular pulses of width 1 nanosecond at a repetition rate of 1 kHz. If the average power output is 1mW, the average power over a single pulse duration, in watts, is  
 (A) 1 (B) 10 (C) 100 (D) 1000

**Key: (D)**

**Sol:** The nature of waveform will be



→ It is given that frequency = 1 kHz, so its period is 1 m.sec.

→ Energy of each pulse,  $E = \frac{P_{av}}{\text{frequency}} = \frac{1 \times 10^{-3}}{1 \times 10^3} = 1 \times 10^{-6}$  joules

→ Power of each pulse,  $P_{pulse} = \frac{E}{\text{Duration of pulse}} = \frac{1 \times 10^{-6}}{1 \times 10^{-9}} = 1000$

49. The dynamics of the state  $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$  of a system is governed by the differential equation

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -3 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 20 \\ 10 \end{bmatrix}$$

Given that the initial state is  $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ , the steady state value of  $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$  is

- (A)  $\begin{bmatrix} -30 \\ -40 \end{bmatrix}$  (B)  $\begin{bmatrix} -20 \\ -10 \end{bmatrix}$  (C)  $\begin{bmatrix} 5 \\ -15 \end{bmatrix}$  (D)  $\begin{bmatrix} 50 \\ -35 \end{bmatrix}$

**Key: (D)**

**Sol:**  $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -3 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 20 \\ 10 \end{bmatrix}$  and initial state is  $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$$x(t) = L^{-1} \left\{ [sI - A]^{-1} X(0) \right\} + L^{-1} \left\{ [sI - A]^{-1} B(s) V(s) \right\}$$

As initial state is zero,  $x(t)$  reduced to  $x(t) = L^{-1} \left\{ [sI - A]^{-1} B(s) V(s) \right\}$

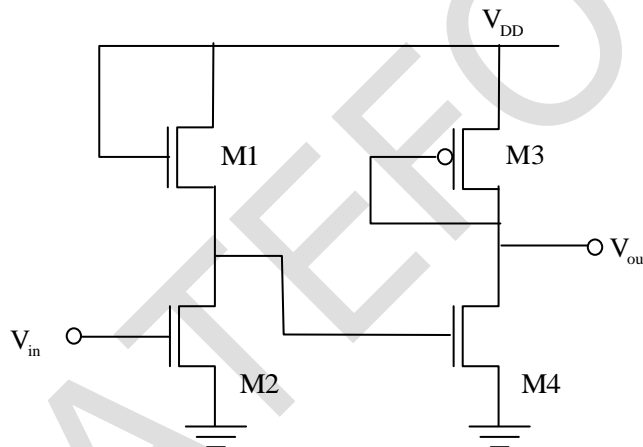
$$(sI - A)^{-1} = \begin{bmatrix} s-1 & 2 \\ -3 & s+4 \end{bmatrix}^{-1} \Rightarrow \frac{1}{(s+4)(s-1)+6} \begin{bmatrix} s+4 & +2 \\ -3 & (s-1) \end{bmatrix}$$

$$sX(s) = \frac{1}{(s+4)(s-1)+6} \begin{bmatrix} s+4 & 2 \\ -3 & s+1 \end{bmatrix} \begin{bmatrix} 20 \\ 10 \end{bmatrix}$$

$$\lim_{s \rightarrow 0} X(s) = \frac{1}{2} \begin{bmatrix} 4 & 2 \\ -3 & -1 \end{bmatrix} \begin{bmatrix} 20 \\ 10 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 80+20 \\ -60-10 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 100 \\ -70 \end{bmatrix} = \begin{bmatrix} 50 \\ -35 \end{bmatrix}$$

Hence, the steady state value of  $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$  is  $\begin{bmatrix} 50 \\ -35 \end{bmatrix}$

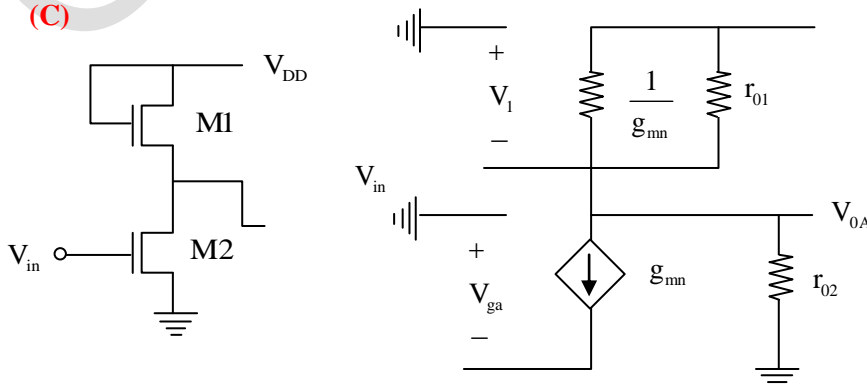
50. A Voltage amplifier is constructed using enhancement mode MOSFETs labeled M1, M2, M3 and M4 in the figure below. M1, M2 and M4 are n-channel MOSFETs and M3 is a p-channel MOSFET. All MOSFETs operate in saturation mode and channel length modulation can be ignored. The low frequency, small signal input and output voltages are  $V_{in}$  and  $V_{out}$  respectively and the dc power supply voltage is  $V_{DD}$ . All n-channel MOSFETs have identical transconductance  $g_{mn}$  while the p-channel MOSFET has transconductance  $g_{mp}$ . The expressions for the low frequency small signal voltage gain  $V_{out}/V_{in}$  is



- (A)  $-g_{mn} / g_{mp}$  (B)  $-g_{mn} (g_{mn} + g_{mp})^{-1}$   
 (C)  $+g_{mn} / g_{mp}$  (D)  $g_{mn} (g_{mn} + g_{mp})^{-1}$

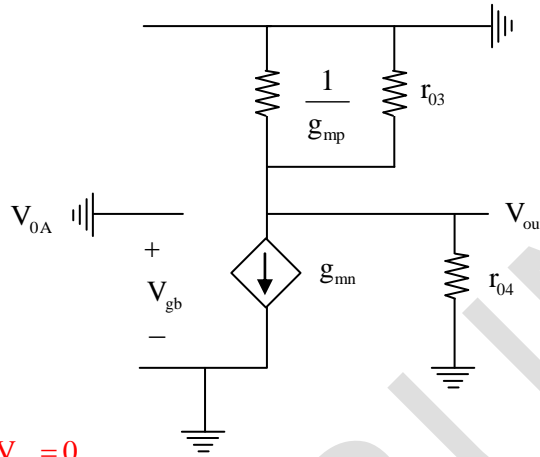
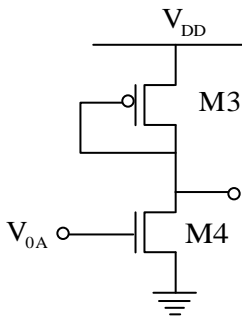
**Key:** (C)

**Sol:**



$$V_o + g_{mn} \left[ \gamma_{01} \parallel \gamma_{02} \parallel \frac{1}{g_{mn}} \right] V_{ga} = 0 \quad \because \gamma_{01} = \gamma_{02} = \infty$$

$$\frac{V_{0A}}{V_{in}} = -\frac{g_{mn}}{g_{mn}} = -1 \quad \dots (1)$$



$$V_{out} + g_{mn} \left[ \gamma_{03} \parallel \gamma_{04} \parallel \frac{1}{g_{mp}} \right] V_{gb} = 0$$

$$\because V_{gb} = V_{0A} \text{ and } \gamma_{03} = \gamma_{04} = \infty$$

$$\frac{V_{out}}{V_{0A}} = -\frac{g_{mn}}{g_{mp}} \quad \dots (2)$$

$$\frac{V_{out}}{V_{in}} = \frac{V_{0A}}{V_{in}} \times \frac{V_{out}}{V_{0A}} = (-1) \left[ \frac{-g_{mn}}{g_{mp}} \right]$$

$$\frac{V_{out}}{V_{in}} = \frac{g_{mn}}{g_{mp}}$$

51. The transfer function relating the input  $x(t)$  to the output  $y(t)$  of a system is given by  $G(s) = 1/(s+3)$ . A unit-step input is applied to the system at time  $t=0$ . Assuming that  $y(0) = 3$ , the value of  $y(t)$  at time  $t = 1$  is \_\_\_\_\_. (Rounded off to two decimal places).

**Key:** (0.46)

**Sol:** It is given that

$$G(s) = \frac{1}{s+3}$$

$$\Rightarrow \frac{Y(s)}{X(s)} = \frac{1}{s+3} \Rightarrow SY(s) + 3Y(s) = X(s)$$

$$\Rightarrow \frac{dy(t)}{dt} + 3y(t) = x(t)$$

[Since initial condition is specified, to incorporate it we need to derive its corresponding differential equation]

Taking Laplace on both sides

$$sY(s) - y(0) + 3Y(s) = X(s)$$

$$\begin{aligned} \Rightarrow Y(s) &= \frac{x(s)}{s+3} + \frac{y(0)}{s+3} \\ &= \frac{1}{s(s+3)} + \frac{3}{s+3} \quad \left[ \because \text{given not } x(t) = u(t) \Rightarrow x(s) = \frac{1}{s} \text{ and } y(0) = 1 \right] \\ &= \frac{1/3}{s} + \frac{-1/3}{s+3} + \frac{3}{s+3} = \frac{1/3}{s} + \frac{8/3}{s+3} = \frac{1}{3} \left[ \frac{1}{s} + \frac{8}{s+3} \right] \end{aligned}$$

$$\Rightarrow y(t) = \frac{1}{3} [1 + 8e^{-3t}] u(t)$$

$$\begin{aligned} \Rightarrow y(t) &= \frac{1}{3} [1 + 8e^{-3 \cdot 1}] u(1) \\ &= \frac{1}{3} [1 + 8e^{-3}] = 0.466 \quad \left[ \because u(t) = 1 \right] \end{aligned}$$

52. In a control system with unit gain feedback, the plant has the transfer function  $P(s) = 3/s$ . Assuming that a controller of the form  $C(s) = K/(s+9)$  is used, where  $K$  is a positive constant, the value of  $p$  for which the root-locus of the closed-loop system passes through the points  $-3 \pm j3\sqrt{3}$  where  $j = \sqrt{-1}$ , is

- (A) 3                      (B)  $3\sqrt{3}$                       (C) 6                      (D) 9

**Key:** (C)

**Sol:** Given, plant transfer function  $= \frac{3}{s}$

$$\text{Controller } (s) = \frac{k}{(s+p)}$$

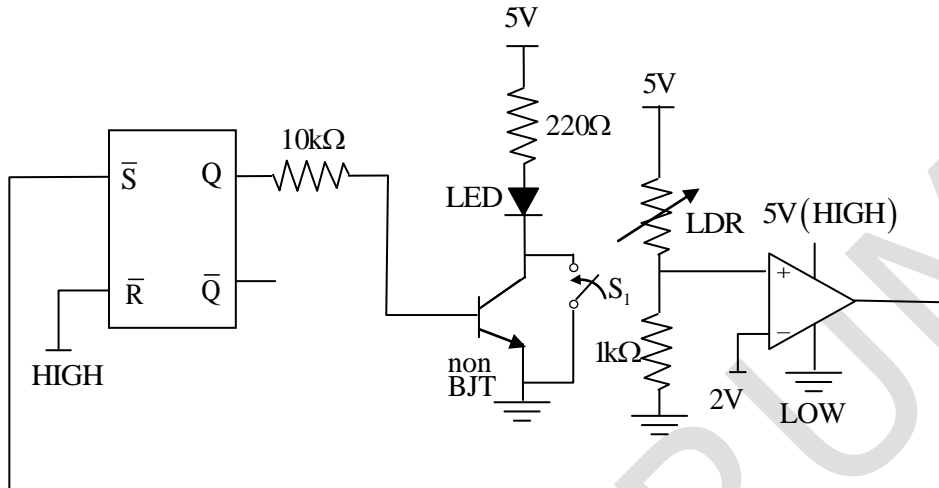
$$G(s)H(s) = C(s)P(s)H(s) = \frac{3k}{s(s+p)} \quad (\because H(s) = 1)$$

As the root focus of the closed-loop system passes through the point  $-3 \pm j3\sqrt{3}$ . Angle condition should following. Angle condition stated as for a point to lie on root locus, the angle evaluated at the point must be an odd multiple of  $\pm 180^\circ$ .

$$\begin{aligned} \angle G(s)H(s) &= -90^\circ - \tan^{-1} \left\{ \frac{3\sqrt{3}}{P-3} \right\} \\ &= -90^\circ - \tan^{-1} \left\{ \frac{3\sqrt{3}}{P-3} \right\} = -180 = \tan^{-1} \left( \frac{3\sqrt{3}}{P-3} \right) = 90^\circ \end{aligned}$$

$$P - 6 = 0 \Rightarrow P = 6$$

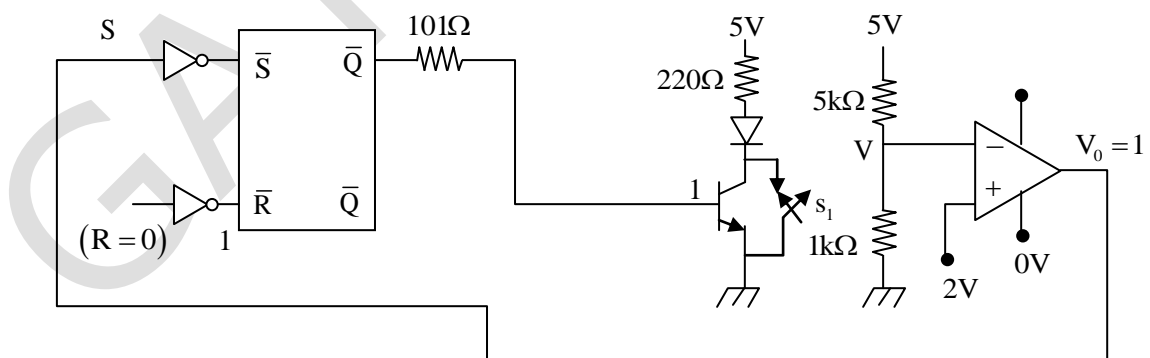
53. In the circuit below, the light dependent resistor (LDR) receives light from the LED. The LDR has resistances of  $5k\Omega$  and  $500\Omega$  under dark and illuminated conditions, respectively. The LED is OFF at time  $t < 0$ . At time  $t = 0s$ , the switch  $S_1$  is closed for 1ms and then kept open thereafter. Assuming zero propagation delay in the devices, the LED



- (A) turns ON when  $S_1$  is closed and remains ON after  $S_1$  is opened.  
 (B) turns ON when  $S_1$  is closed and turns OFF after  $S_1$  is opened.  
 (C) turns ON when  $S_1$  is closed and toggles periodically from ON to OFF after  $S_1$  is opened.  
 (D) remains OFF when  $S_1$  is closed and continues to remain OFF after  $S_1$  is opened.

**Key:** (A)

**Sol:** It is given that, for  $t < 0$   
 Switch  $S_1$ , is open and LED is off



Since LED is off for  $t < 0$ , the resistance of LDR under dark condition is  $5000\Omega = 5k\Omega$

$$\text{So } V^- = \left[ \frac{1000}{5000 + 1000} \right] 5 = \frac{5}{6} = 0.533$$

$$V^+ = 2V \quad \therefore V^+ > V^- \Rightarrow V_0 = 1 = S$$

Now to the SRFF,  $S = 1, R = 0$

So  $Q^+ = 1$

For  $0 < t < 1$  m-sec, Switch  $S$ , is closed,

So LED will be F. B and it will glow, now the resistance of LDR will be  $500\Omega$

$$V^- = \frac{1000}{500+1000} \times 5 = \frac{5000}{1500} = 3.33V$$

Now  $V^- > V^+ \Rightarrow V_0 = 0 = S$

Now  $S=0, R=0$ , So  $Q^+ = Q(t < 0) = 1$

When  $Q^+ = 1$ , the base of npn transistor is 1. So the transistor will behave as a on switch.

So LED will glow.

So during  $0 < t < 1$  m sec,  $S_1$  is closed

$Q = 1 \Rightarrow$  Transistor, ON

And  $V_0 = 1$

For  $t > 1$  m-sec,  $S_1$  open, even the switch  $S_1$  is open, still  $Q = 1$ , Transistor is ON, LED will glow for ever.

So the LED, turns ON when  $S_1$  is closed and remain ON after  $S_1$  is opened.

54. A complex function  $f(z) = u(x,y) + iv(x,y)$  and its complex conjugate  $f'(z) = u(x,y) - i v(x,y)$  are both analytic in the entire complex plane, where  $z = x + iy$  and  $i = \sqrt{-1}$ . The function  $f$  is then given by

(A)  $f(z) = x + iy$

(B)  $f(z) = x^2 - y^2 + i 2xy$

(C)  $f(z) = \text{constant}$

(D)  $f(z) = x^2 + y^2$

**Key:** (C)

**Sol:** Let  $f(z) = k$  (constant may be real or complex)

And  $f^*(z) = u - iv$  is either  $k$  or  $\bar{k}$ , which is also constant

Also no other option satisfied that both  $f(z)$  and  $f^*(z)$  are analytic.

55. The output of a continuous-time system  $y(t)$  is related to its input  $x(t)$  as

$$y(t) = x(t) + \frac{1}{2}x(t-1). \text{ If the Fourier transforms of } x(t) \text{ and } y(t) \text{ are } X(\omega) \text{ and } Y(\omega)$$

respectively, and  $|X(0)|^2 = 4$ . The value of  $|Y(0)|^2$  is \_\_\_\_\_,

**Key: (9)**

**Sol:** It is given that

$$y(t) = x(t) + \frac{1}{2}x(t-1) \text{ and } |x(0)|^2 = 4$$

We need to obtain  $|Y(0)|^2$

$$y(t) = x(t) + \frac{1}{2}x(t-1)$$

$$Y(\omega) = X(\omega) + \frac{1}{2}X(\omega)e^{-j\omega} = \left(1 + \frac{1}{2}e^{-j\omega}\right)X(\omega)$$

$$Y(0) = \left(1 + \frac{1}{2}e^{-j0}\right)X(0) = \frac{3}{2}X(0)$$

$$|Y(0)| = \frac{3}{2}|X(0)|$$

$$|Y(0)|^2 = \left(\frac{3}{2}\right)^2 |X(0)|^2 = \frac{9}{4} \times 4 = 9$$



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